Chiral Symmetry Breaking
from Monopoles and Duality

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Motivation

Confinement and chiral symmetry breaking are well established

Goal: Find deformations of QCD, continuously connected to the full theory, that exhibit $\chi$SB and confinement.

Bali et al. (2000), Borsanyi et al (2011)
Consider $SU(2)$ gauge theory with $N_f^{ad} = 1$ on $R^3 \times S_1$

$$\mathcal{L} = -\frac{1}{4g^2} F_{\mu\nu}^a F^{a \mu\nu} - \frac{i}{g^2} \lambda^a \sigma \cdot D^{ab} \lambda^b + \frac{m}{g^2} \lambda^a \lambda^a$$

$$A_\mu^a(0) = A_\mu^a(L)$$

Large mass limit: Pure YM. Small mass limit: SUSY YM.

Small $S_1$ and $m$: Confinement can be studied using semi-classical methods, based on monopole-instantons, instantons, and bions.

Low energy fields: Holonomy $b$ and dual photon $\sigma$
Non-perturbative effects

Topological classification on $R^3 \times S_1$ (GPY)

1. Topological charge

$$Q_{top} = \frac{1}{16\pi^2} \int d^4x \ F\tilde{F}$$

2. Holonomy (eigenvalues $q^\alpha$ of Polyakov line at spatial infinity)

$$\langle \Omega(\vec{x}) \rangle = \langle \text{Tr exp} \left[ i \int_0^\beta A_4 dx_4 \right] \rangle$$

3. Magnetic charges

$$Q^\alpha_M = \frac{1}{4\pi} \int d^2S \text{Tr} [P^\alpha B]$$
Periodic instantons (calorons)

Instanton solution in $R^4$ can be extended to solution on $R^3 \times S^1$

$SU(2)$ solution has $1 + 3 + 1 + 3 = 8$ bosonic zero modes

$$2S_0 = \frac{8\pi^2}{g^2}$$

$4n_{adj}$ fermionic zero modes

$$\int d^2 \zeta d^2 \xi$$
KvBLL (1998) construct calorons with non-trivial holonomy

BPS and KK monopole constituents. Fractional topological charge, 1/2 at center symmetric point.

\[ 2 \times (3 + 1) = 8 \text{ bosonic zero modes}, \ 2 \times 2 \text{ fermionic ZM}. \]

\[ \int d\phi_1 \int d^3 x_1 \int d^2 \zeta e^{-S_1} \int d\phi_2 \int d^3 x_2 \int d^2 \xi e^{-S_2} \]
Topological objects

\[(Q_M, Q_{top}) = (\int_{S^2} B \cdot d\Sigma, \int_{R^3 \times S^1} F \tilde{F})\]

BPS \[\begin{array}{c}
\begin{array}{c}
\uparrow \\
\downarrow \\
\end{array}
\end{array}\]

KK \[\begin{array}{c}
\begin{array}{c}
\downarrow \\
\uparrow \\
\end{array}
\end{array}\]

monopoles

\[(1, 1/2) \quad (-1, 1/2) \quad (-1, -1/2) \quad (1, -1/2)\]

BPS \[\begin{array}{c}
\begin{array}{c}
\uparrow \\
\downarrow \\
\end{array}
\end{array}\]

KK \[\begin{array}{c}
\begin{array}{c}
\downarrow \\
\uparrow \\
\end{array}
\end{array}\]

instantons

\[(0, 1) \quad (0, -1)\]

BPS \[\begin{array}{c}
\begin{array}{c}
\uparrow \\
\downarrow \\
\end{array}
\end{array}\]

KK \[\begin{array}{c}
\begin{array}{c}
\downarrow \\
\uparrow \\
\end{array}
\end{array}\]

bions

\[(0, 0) \quad (0, 0)\]

Note: BPS/KK topological charges in \(Z_2\) symmetric vacuum. Also have \((2, 0)\) (magnetic) bions.
Effective potential

Instantons and monopoles: Exact solutions, but $V(b, \sigma) = 0$.

Bions: Approximate solutions

$$V_{BPS,BPS} \sim e^{-2b} e^{-2S_0} \int d^3r e^{-S_{12}(r)}$$

$$S_{12}(r) = \frac{4\pi L}{g^2 r} (q_1^m q_2^m - q_1^b q_2^b) + 4 \log(r)$$

Saddle point integral after analytic continuation $g^2 \rightarrow -g^2$ (BZJ)

$$V(b, \sigma) \sim \frac{M_{PV}^6 L^3 e^{-2S_0}}{g^6} \left[ \cosh \left( \frac{8\pi}{g^2} (\Delta \theta - \pi) \right) - \cos(2\sigma) \right]$$

Center symmetric vacuum $\text{tr}(\Omega) = 0$ preferred

Mass gap for dual photon $m_\sigma^2 > 0$ ($\rightarrow$ confinement)
What about chiral symmetry breaking?

Original setup: One adjoint fermion, chiral symmetry is discrete.

\[ \langle \lambda \lambda \rangle \neq 0 \quad Z_{2N_c} \to Z_2 \]

Light fundamental fermions: Need strong coupling.

\[ \mathcal{L} \sim G \det_{N_f} (\bar{\psi}_L \psi_R) + \text{h.c.} \]

Heavy fundamental fermions: Study explicit breaking of \( Z_N \) center symmetry.
Role of Boundary Conditions

Consider flavor twisted boundary conditions

\[ \psi(\tau + \beta) = \Omega_F \psi(\tau) \quad \Omega_F = \text{diag}(1, e^{2\pi i/N_f}, \ldots, e^{2\pi i(N_f-1)/N_f}) \]

Flavor holonomy \( \Omega_F \) has several interesting properties:

1. \( N_f = N_c \): Respects \( Z_{N_c} \) center symmetry.
2. Large \( L \): Breaks flavor symmetry, but in a controlled fashion.
Large L expectations

Flavor holonomy corresponds imaginary flavor (isospin) chemical potential $\tilde{\mu}_F \sim i/L$.

Can be studied using chiral Lagrangian

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr} \left[ \nabla_\mu U \nabla^\mu U^\dagger \right] - B \text{Tr} \left[ M U + h.c \right]$$

with $\nabla_\mu U = \partial_\mu U + i[\tilde{\mu}_F T_F, U]$.

Consider $N_f = 2$ (isospin chemical potential)

$$m_{\pi^0}^2 = m_\pi^2, \quad m_{\pi^\pm}^2 = m_\pi^2 + \tilde{\mu}_I^2$$

$N_f - 1$ exact Goldstone modes ($m=0$), others acquire gaps.
Small L theory: Perturbation theory

Consider center symmetric gauge holonomy (add double trace deformation). For $l \gtrsim N_c L$ theory abelianizes

$$SU(N_c) \rightarrow [U(1)]^{N_c - 1}$$

Gapless (Cartan) gluons described by dual photon $\vec{\sigma}$

$$S = \frac{g^2}{8\pi^2 L} \int d^3 x (\partial_\mu \vec{\sigma})^2$$

with $F_{\mu\nu}^i = \frac{g^2}{2\pi L} \epsilon_{\mu\nu\alpha} \partial^\alpha \sigma^i$.

Remain gapless to all orders in perturbation theory due to emergent shift symmetry $\vec{\sigma} \rightarrow \vec{\sigma} + \vec{\epsilon}$.
Small L theory: Semiclassical objects

Center symmetric background, no fermions: Instanton fractionalize into $N_c$ constituents

$$M_i \sim e^{-S_0} e^{i\tilde{\alpha}_i \cdot \vec{\sigma}} \quad S_0 = \frac{8\pi^2}{g^2 N_c} \quad \tilde{\alpha}_i \quad SU(N_c) \text{ root vectors}$$

In the ground state these objects proliferate: The monopole-anti-monopole gas.

$$V(\vec{\sigma}) \sim m_W^3 e^{-S_0} \sum_i \cos(\tilde{\alpha}_i \cdot \vec{\sigma})$$

Mass gap for the dual photon, continuous shift symmetry broken.

Massless fermions: Take into account fermion zero modes.
Small L theory: Fermion zero modes

Many eigenvalue circles: Polyakov line  Flavor holonomy

Instanton-monopoles  \( \theta \) flavor singlet twist

\[
N_c = N_f = 4 \quad N_c = 4 \quad N_f = 3
\]

Zero modes localize on monopoles jumping over flavor eigenvalues

Bruckmann, Nogradi, van Baal (2003); Moore et al. (2014)
Two basic scenarios \((N_c = N_f)\)

No flavor twist: Standard 't Hooft vertex carried by one monopole

\[
\mathcal{M}_1 \sim e^{-S_0} e^{i\vec{\alpha}_1 \cdot \vec{\sigma}} \det_F(\bar{\psi}_L^f \psi_R^g) \quad \mathcal{M}_{i>1} \sim e^{-S_0} e^{i\vec{\alpha}_i \cdot \vec{\sigma}}
\]

Center symmetric flavor holonomy: Single flavor 't Hooft vertex carried by each monopole

\[
\mathcal{M}_i \sim e^{-S_0} e^{i\vec{\alpha}_i \cdot \vec{\sigma}} (\bar{\psi}_L^i \psi_R^i)
\]

![Diagram of monopoles and flavor holonomies](image)
Spontaneous symmetry breaking

Unbroken symmetries of flavor twisted theory

\[ [U(1)_J]^{N_c-1} \times [U(1)_V]^{N_f-1} \times [U(1)_A]^{N_f-1} \times U(1)_Q \]

Shift symmetry \hspace{1cm} \text{Exact flavor symmetry}

Symmetries of monopole vertex

\[ \mathcal{M}_i \sim e^{-S_0} e^{i\vec{\alpha}_i \cdot \vec{\sigma}} (\bar{\psi}_L^i \psi_R^i) \]

Preserves vectorial symmetry \([U(1)_V]^{N_f-1} \times U(1)_Q\). Breaks axial symmetry

\[ [U(1)_A]^{N_f-1} : \quad (\bar{\psi}_L^f \psi_R^f) \rightarrow e^{i\epsilon_i} (\bar{\psi}_L^i \psi_R^i) \]
Spontaneous symmetry breaking, continued

Monopole vertex is invariant provided $[U(1)_A]^{N_f-1}$ is combined with $[U(1)_J]^{N_c-1}$ shift symmetry

$$[\tilde{U}(1)_A]^{N_f-1} : \begin{cases} (\bar{\psi}_L^f \psi_R^f) \rightarrow e^{i\epsilon_i} (\bar{\psi}_L^i \psi_R^i) \\ e^{i\alpha_i \cdot \vec{\sigma}} \rightarrow e^{-i\epsilon_i} e^{i\alpha_i \cdot \vec{\sigma}} \end{cases}$$

Ground state $\langle e^{i\alpha_i \cdot \vec{\sigma}} \rangle \rightarrow 1$. Breaks

$$[U(1)_V]^{N_f-1} \times [\tilde{U}(1)_A]^{N_f-1} \rightarrow [U(1)_V]^{N_f-1}$$

For $m = 0$ the ground state is degenerate. Massless Goldstone boson

$$S_\sigma = L \int d^3x \left\{ \frac{f^2}{4} \text{Tr} \left[ \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] - B \text{Tr} \left[ M \Sigma + h.c. \right] \right\}$$

Microscopically $\Sigma = e^{i\Pi/f_\pi}$ with $\Pi = \pi^a T^a$ and $\pi^a = \frac{g}{2\pi L} \sigma^a$

Color-flavor transmutation
Chiral Lagrangian

Chiral lagrangian has calculable coefficients

\[ S_\sigma = L \int d^3 x \left\{ \frac{f_\pi^2}{4} \text{Tr} \left[ \partial_\mu \Sigma \partial^\mu \Sigma^\dagger \right] - B \text{Tr} \left[ M \Sigma + h.c. \right] \right\} \]

\[ f_\pi^2 = \left( \frac{g}{\sqrt{6\pi L}} \right)^2 = \frac{N_c \lambda m_W^2}{24\pi^2} \]

\[ B = -\frac{1}{2} \langle \bar{\psi} \psi \rangle \sim m_W^{-3} e^{-\frac{8\pi^2}{\lambda}} \]

Also note: VEV of monopole operator can be viewed as effective constituent quark mass

\[ m_Q \sim m_W e^{-\frac{8\pi^2}{\lambda}} \]
Conclusions and Outlook

Calculable mechanism for chiral symmetry breaking in a compactified version of QCD.

Results consistent with continuity between large $L$ (full QCD) and small $L$ theory.

Mechanism based on monopole instantons and color flavor transmutation.

Study: Extension to $N_f > N_c$? Relation between $\chi$SB and confinement?