Physics Problem Solving and Metacognition – different descriptions by different instructors*

Vince H. Kuo
North Carolina State University

Patricia Heller & Ken Heller
University of Minnesota
Charles Henderson
Western Michigan University
Edit Yerushalmi
Weizmann Institute

http://www.ncsu.edu/per/
*Supported in part by NSF grant #DUE-9972470
Outline of talk

1. Putting the study into perspective
2. What did we do?
3. Solving physics problems
4. Metacognition
5. Implications
Grand Scheme of Things …

Curricular Material that physics instructors will adopt for their introductory calculus-based physics courses

Instructors’ conceptions about various elements in the educational environment of introductory calculus-based physics

Exploratory Model of instructors’ conceptions about the teaching and learning of problem solving

Instructors’ conceptions about the problem-solving process
What did we do?

As previously reported*, the study consists of:

- Interview tool based on instructional artifacts focused around a single problem
  - 3 Instructor Solutions
  - 5 Student Solutions
  - 4 Problem Types
- General & Specific Questions

- Sample of 30 physics instructors from 4 types of higher education institutions in the state of Minnesota
  - Research University (6)
  - State University (8)
  - Private College (9)
  - Community College (7)

Generate hypotheses to be tested with larger sample!

What did we do?

As previously reported*, the study consists of:

• A targeted analysis:
  – **Identify** parts of interview where statements about the problem-solving process were found in previous study
    • based on results from extensive analysis of interview with research university instructors
  – **Code statements relevant to students’ problem-solving process into**
    • Mechanical
    • Procedural
    • **Metacognitive**

Students should think about how visualizing the problem situation helps them organize the information

What is Problem Solving?

“Process of Moving Towards a Goal When the Path is Uncertain”
- If you know how to do it, it is not a “problem”
- A problem for a student is not a problem for the faculty

**Exercise vs. Problem**

- **Problems are solved using tools**
  - General Purpose Heuristics

- **Problem Solving involves Uncertainty and Mistakes**

M. Martinez (1998), *Phi Beta Kappan*, April
What is Problem Solving?

Limitation: Instructors’ conceptions are inferred from what they talk about when describing the problem-solving process during the interview, in the context of introductory calculus-based physics, not about how they actually solve problems or how they actually teach.

Solving Physics Problems can be characterized as a decision-making process that involves linear, cyclical, and artistic problem-solving approaches.

1. **Linear (100%, n = 22)**
   - Understanding the problem (41%)
   - Visualization, extraction, and categorization of the physical situation (59%)
   - Decide on where to start (73%)
   - Recognize, decide on, and list the principles and concepts needed (82%)
   - Apply the principles and concepts (64%)
   - Plug the numbers into the equations (32%)
   - Answer (41%)

2. **Cyclical (100%, n = 7)**
   - Listing, labeling, and defining all relevant variables (73%)
   - Drawing pictures and diagrams (95%, Linear, 86% Cyclical)
   - Having an understanding of physics principles and concepts (77%, Linear, 57% Cyclical)
   - Equations written in symbolic form (41%)
   - Pay attention to units and dimensions (41%)
   - Make assumptions when necessary (36%)
   - Checking the units (36%, Linear, 29% Cyclical)
   - Evaluating the reasonableness (55%, Linear, 29% Cyclical)
   - Answer (41%)

3. **Artistic (100%, n = 1)**
   - Understanding, focusing, visualizing, and analyzing the problem (71%)
   - Brainstorm and explore to come up with possible approaches (100%)
   - Experiment on an approach (57%)
   - Figuring out what is needed (57%)
   - Solve for what is being asked (43%)
   - Apply the principles and concepts (86%)
   - Go through the mathematics (43%)
   - Evaluate the answer (57%)

Each step requires careful consideration and can be revisited if necessary.
1. Decision-Making Process – **Linear** *(n = 22)*

**Step 1:** Understand the problem

**Step 2:** Visualize, extract, & categorize the physical situation

**Step 3:** “Know” the correct physics principle(s) & the approach based on experience of having solved many problems

**Step 4:** Apply the principle(s) & make assumptions when necessary

**Step 5:** Go through the mathematics

**Step 6:** Evaluate the answer
2. Decision-Making Process - Cyclical (n = 7)

**Step 1:** Understand, focus, & visualize the problem

**Step 2:** “Brainstorm” & “Explore” to come up with possible approaches & principle(s)

**Step 3:** “Experiment” on an approach by deciding on where to start & apply the principle(s) (go back to Step 2 as necessary)

**Step 4:** Go through the mathematics (go back to Step 2 or Step 3 as necessary)

**Step 5:** Evaluate the answer (go back to Step 2 as necessary)
What is Metacognition?

- People’s Thinking about their own Thinking


Underlies all higher order thinking, especially problem solving!
Why do we care?

• Research has indicated (Schoenfeld, 1983, 1985a, 1985b, 1987; Lester, Garofalo, & Kroll, 1989)
  – Successful problem solvers spend more time
    … planning the directions that may be taken
    … monitor and evaluate their actions and
    cognitive processes throughout problem-solving episodes than do less successful
    problem solvers
  – Problem-Solving Frameworks that explicitly
    emphasize metacognitive processes can be
    effective in teaching problem solving

Let’s see how these instructors view the role of
metacognition
What are we doing?

Separating metacognitive statements into

Planning
Statements related to starting a solution to a problem

Monitoring
Statements related to checking the progress of solution

Evaluating
Statements related to checking the reasonableness of solution
Metacognition — Previously Reported*

- Range of meta/PS statements ~ 5% to ~ 50%
- No dependence with Type of Institution
- No correlation with Contact Hours with students
- Significant correlation with Teaching Experience
  - $R = -0.51$ (p < 0.01)

* AAPT Conference Presentations (Winter 2004)
A naïve assumption could be that these instructors would consider *planning*, *monitoring*, and *evaluating* equally in problem solving.

<table>
<thead>
<tr>
<th>Count</th>
<th>Problem Solving</th>
<th>Metacognition</th>
<th>Planning</th>
<th>Monitoring</th>
<th>Evaluating</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>14</td>
<td>7</td>
<td>3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Max</td>
<td>116</td>
<td>53</td>
<td>31</td>
<td>21</td>
<td>8</td>
</tr>
<tr>
<td>Average</td>
<td>65</td>
<td>20</td>
<td>12</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>1948</td>
<td>606</td>
<td>360</td>
<td>172</td>
<td>74</td>
</tr>
</tbody>
</table>

\[ H_0: n_p = n_m = n_e \quad \Rightarrow \quad \chi^2 = 209.15, \ p < 0.000 \]

These instructors, as a whole, did not talk about the three types of *metacognition* equally when describing the problem-solving process.
Metacognition – Different Types

Metacognitive Type vs. Type of Problem-Solving Process

- Linear
  - 2 Items: Evaluating (3 Items), Monitoring (7 Items), Planning (18 Items)
  - 5 Items: Evaluating (4 Items), Monitoring (11 Items), Planning (24 Items)

- Cyclical
  - 3 Items: Evaluating (4 Items), Monitoring (10 Items), Planning (18 Items)
  - 5 Items: Evaluating (11 Items), Monitoring (24 Items), Planning (18 Items)
Metacognition – Example Comparison

**Linear**

- Know to think about how to best approach the problem (32%)
  - Decide on where to start (73%)
    - and then
  - Recognize, decide on, and list the principles and concepts needed (82%)
    - then
  - Apply the principles and concepts (64%)

**Cyclical**

- Know to think about how to best approach the problem (57%)
  - Brainstorm and explore to come up with possible approaches (100%)
    - and
  - Know to brainstorm, splatter, and explore ideas about how to best approach the problem (57%)
    - Know to think about what one is doing to set up an organized plan of steps (71%)
    - Realize when the solution is not progressing desirably (86%)
      - Know to evaluate the progress of the solution (30%)
      - Know to check the process of the solution (30%)
    - Know to think explicitly about and justify reasoning that goes into the steps of a solution (30%)
      - Experiment on an approach (57%)
Questions Raised

From this small sample

1. Do the different conceptions of the problem-solving process contribute to the lack of adoption of research-based curricula by instructors?

2. Do the different conceptions of the metacognitive processes contribute to the lack of adoption of research-based curricula by instructors?
Implications — Practical

Research has shown that problem-solving frameworks that embody metacognitive processes can be effective tools in the instruction of problem solving.

It is unclear, however, if physics instructors, as experts in the field, can adequately unpack the internalized knowledge on their own so as to make the instruction on problem solving and metacognition explicit and coherent.

1. Instructors expressed conceptions similar to the problem-solving frameworks in literature (with different words and number of steps)
   - Frameworks and instructional structures must be flexible so instructors have the freedom to refine as they see fit.
   - Frameworks and instructional structures must be robust so that the refinements are not counter-productive.
Implications – Practical

Research has shown that problem-solving frameworks that embody metacognitive processes can be effective tools in the instruction of problem solving.

It is unclear, however, if physics instructors, as experts in the field, can adequately unpack the internalized knowledge on their own so as to make the instruction on problem solving and metacognition explicit and coherent.

2. Instructors expressed limited conceptions about certain metacognitive processes (not as much monitoring and evaluating as planning)

- Frameworks and instructional structures must explicitly address all metacognition, and provide language with which to frame such metacognition during instruction.
- Frameworks and instructional structures must provide opportunities for instructors to experience the positive effects of making metacognitive processes explicit.
Next Steps …

• Close-ended survey/questionnaire for determining the distribution of physics instructors’ conceptions in national sample
  – Conceptions on process
  – Conceptions on decision making
  – Main units of process
  – Detail of process
  – Role of Metacognition
  – Types of Metacognition
  – etc …
Thanks Everybody!

Please visit our website for more information:
http://www.ncsu.edu/per/

Or send Email to:
hvkuo@ncsu.edu
The Interview Tool

To investigate instructor conceptions, we developed a 1½ - 2 hour interview based on instructional artifacts:

1st) 3 Instructor solutions: varied in the details of their explanation, physics approach, and presentation structure

2nd) 5 Student solutions: based on actual final examination solutions at the University of Minnesota to represent features of student practice

3rd) 4 Problem types: represent a range of the types of problems used in introductory physics courses

All artifacts were based on one problem -- instructors were given the problem and asked to solve it on their own before the interview
The Interview Tool - IS

**Instructor solution I**

The stone only does work:

Conservation of energy between points A and B:

\[ \text{KE}_A = \text{KE}_B \]

\[ \frac{1}{2}mv^2 = \text{mg}h \]

At points A, non-kinetic: 0 kWh

\[ \text{PE} = \text{KE} \]

\[ T - w = m\text{g}h \]

\[ E_{\text{top}} = E_{\text{bottom}} \]

\[ \text{KE} = \frac{1}{2}mv^2 = \text{mg}h \]

\[ v^2 = 2gh \]

\[ h = \frac{v^2}{2g} \]

\[ h = \frac{v^2}{2g} \]

**Instructor solution II**

**Instructor solution III**

\[ \text{Approach:} \]

I need to find \( F_{\text{max}} \), force exerted by me. I know the path, \( h \) (height at top) and \( v \) (velocity at top).

A) For a massless string \( F_{\text{max}} = T_\text{max} \) (Tension at bottom)

B) I can relate \( T_\text{max} \) to \( v \) (velocity at bottom) using the radial component of \( 2F_{\text{max}}\), and radial acceleration of \( v \), since stone is in circular path.

C) I can relate \( v \), to \( v \), using either: I) energy 1) Kinetic and Inertial

D) I can apply work-energy theorem for stone. Path has 2 parts:

- First - circular, earth and rope interacts with stone.
- Second - vertical, earth interacts with stone.

In both parts, only force that does work is weight, since in first part hand is not moving \( 180 \) \( F \) does no work.

**Execution**

B) RELATE \( T_\text{max} \) TO \( v \)

\[ \sum F = \text{ma} \]

\[ \sum F_{\text{exerted}} = \text{ma} \]

\[ T_w = m\text{g}h_{\text{top}} \]

Using \( v \) from above:

\[ v_{\text{bottom}} = \sqrt{2gh} \]

C) RELATE \( T_\text{max} \) TO \( v \)

**Work = KE**

For constant force

\[ F = \text{KE}_{\text{top}} - \text{KE}_{\text{bottom}} \]

\[ F = \text{KE}_{\text{top}} - \text{KE}_{\text{bottom}} \]

\[ \text{Check limit: } T_\text{max} \text{ as } R \text{ for smaller circle III need bigger force, reasonable} \]
The Interview Tool - SS

Student Solution A

\[ \begin{align*}
V^2 &= \frac{a^2}{2} \\
V &= \sqrt{\frac{a^2}{2}} \\
V &= \sqrt{\frac{a^2}{2}} \\
\end{align*} \]

Student Solution B

\[ \begin{align*}
\text{Find velocity to reach height (from Example 2.3)}

V_0^2 &= 2a \left( \frac{V}{g} \right) \\
V &= \sqrt{\frac{2aV}{g}} \\
\end{align*} \]

Student Solution C

\[ \begin{align*}
F &= \frac{2mg}{V^2} \\
F &= \frac{2 \times 22 \text{ kg}}{2.16 \text{ m/s}^2} \\
F &= \text{1237.89 N} \\
\end{align*} \]

Student Solution D

\[ \begin{align*}
\text{Forces: tension between rope and block} \\
F &= mg \\
F &= \frac{2 \times 22 \text{ kg}}{2.16 \text{ m/s}^2} \\
F &= \text{1237.89 N} \\
\end{align*} \]

Student Solution E

\[ \begin{align*}
F &= \frac{m \cdot 2gh}{2} \\
F &= \frac{2 \times 22 \text{ kg} \times 2 \text{ m}}{2} \\
F &= \text{1237.89 N} \\
\end{align*} \]
**The Interview Tool - P**

**Problem A**

A 1.8 kg mass is attached to a frictionless pivot point and is moving in a circle at the end of a 65 cm string. The string breaks when the mass is moving directly upward and the mass rises to a maximum height of 23.0 m. What is the tension in the string one-quarter turn before the string breaks? Assume that air resistance can be neglected.

A) What velocity \( v_e \) must the stone have when released in order to rise to 23 meters above the lowest point in the circle?
B) What velocity \( v_e \) must the stone have when it is at its lowest point in order to have a velocity \( v_e \) when released?
C) What force will you have to exert on the string at its lowest point in order for the stone to have a velocity \( v_e \)?

**Problem B**

You are whirling a stone tied to the end of a string around in a vertical circle having a radius of 65 cm. You wish to whirl the stone fast enough so that when it is released at the point where the stone is moving directly upward it will rise to a maximum height of 23 meters above the lowest point in the circle. In order to do this, what force will you have to exert on the string when the stone passes through its lowest point one-quarter turn before release? Assume that by the time that you have gotten the stone going and it makes its final turn around the circle, you are holding the end of the string at a fixed position. Assume also that air resistance can be neglected. The stone weighs 16 N.

A) 1292 N  
B) 1258 N  
C) 1248 N  
D) 1210 N  
E) None of the Above

**Problem C**

You are working at a construction site and need to get a 3 lb. bag of nails to your co-worker standing on the top of the building (60 ft. from the ground). You don’t want to climb all the way up and then back down again, so you try to throw the bag of nails up. Unfortunately, you’re not strong enough to throw the bag of nails all the way up so you try another method. You tie the bag of nails to the end of a 2 ft. string and whirl the string around in a vertical circle. You try this, and after a little while of moving your hand back and forth to get the bag going in a circle you notice that you no longer have to move your hand to keep the bag moving in a circle. You think that if you release the bag of nails where the string is horizontal to the ground that the bag will go up to your co-worker. As you whirl the bag of nails around, however, you begin to worry that the string might break, so you stop and attempt to decide before continuing. According to the string manufacturer, the string is designed to hold up to 100 lbs. You know from experience that the string is most likely to break when the bag of nails is at its lowest point.

**Problem D**

You are whirling a stone tied to the end of a string around in a vertical circle of radius 2 ft. You wish to whirl the stone fast enough so that when it is released at the point where the stone is moving directly upward it will rise to a maximum height of 23 meters above the lowest point in the circle. In order to do this, what force will you have to exert on the string when the stone passes through its lowest point one-quarter turn before release? Assume that by the time that you have gotten the stone going and it makes its final turn around the circle, you are holding the end of the string at a fixed position. Assume also that air resistance can be neglected.

A) For each point labeled in the diagram, circle the symbol(s) that describe how the speed of the stone is changed.

<table>
<thead>
<tr>
<th>Point</th>
<th>Change in Speed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( v_e )</td>
</tr>
<tr>
<td>B</td>
<td>( v_e )</td>
</tr>
<tr>
<td>C</td>
<td>( v_e )</td>
</tr>
<tr>
<td>D</td>
<td>( v_e )</td>
</tr>
<tr>
<td>E</td>
<td>( v_e )</td>
</tr>
</tbody>
</table>

B) At each point on the diagram, draw and label a vector representing net force at that point.

C) At each point, draw and label a vector to represent all of the forces acting on the stone.
Problem Used in the Interview

You are whirling a stone tied to the end of a string around in a vertical circle having a radius of 65 cm. You wish to whirl the stone fast enough so that when it is released at the point where the stone is moving directly upward it will rise to a maximum height of 23 meters above the lowest point in the circle. In order to do this, what force will you have to exert on the string when the stone passes through its lowest point one-quarter turn before release? Assume that by the time that you have gotten the stone going and it makes its final turn around the circle, you are holding the end of the string at a fixed position. Assume also that air resistance can be neglected. The stone weighs 18 N.

Final examination question (Fall, 1997)
Selecting Instructors for Interviews

Physics instructors in Minnesota (~107 meet selection criteria):

• taught introductory calculus-based physics course in the last 5 years (conducted in Spring of 2000)
• could be visited and interviewed in a single day

Sample Randomly Selected:

30 instructors

(From 35 contacted, 5 declined to be interviewed)

Roughly evenly divided among:

1) Community College (CC) N = 7
2) Private College (PC) N = 9
3) State University (SU) N = 8
4) Research University (RU) N = 6

Interviews were videotaped and the audio portion transcribed:

~ 30 pages of text/interview
Implications – *Theoretical*

1. The initial explanatory model can serve as a productive framework from which to study instructor conceptions in more detail

2. Increased observational reliability (*analysis over smaller segments, articulation of more explicit descriptions of model elements, refinements of model elements, and triangulation of observational support*)
   - provided a means for generalizing over samples in the same population
   - strengthened the refined explanatory model as a more viable model
Implications – *Methodological*

1. Identification of relevant segments of the interview allowed for a more targeted analysis procedure that is the nature of more convergent studies

2. The analysis method made the model elements and interconnections explicit
   - proved to be useful when critiquing and refining
   - provided a transparent way to include reference

3. Specific targeting of problem solving effective in uncovering other implicit conceptions that underlie the process