Enabling Code Diversity for Mobile Radio Channels using Long-Range Fading Prediction

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Abstract—Code diversity integrates space-time coding with beamforming by using a small number of feedback bits to select from a family of space-time codes. Different codes lead to different induced channels at the receiver, where Channel State Information (CSI) is used to instruct the transmitter how to choose the code. Feedback can be combined with sub-optimal low complexity decoding of the component codes to match Maximum-Likelihood (ML) decoding performance of any individual code in the family. It can also be combined with ML decoding of the component codes to improve performance beyond ML decoding performance of any individual code. Prior analysis of code diversity did not take into account the effect of the mobile speed and the delay in the feedback channel. This paper demonstrates the practicality of code diversity in space-time coded systems by showing that performance gains based on instantaneous feedback are largely preserved when long-range prediction of time-varying correlated fading channels is employed to compensate for the effect of the feedback delay. To maintain prediction accuracy for realistic SNR, noise reduction that employs oversampled pilots is used prior to fading prediction. We also propose a robust low pilot rate method that utilizes interleaving to improve the spectral efficiency. Simulations are presented for two channel models: the conventional Jakes model and a realistic physical channel model where the parameters associated with the reflectors vary in time and the arrival rays have different strengths and asymmetric arrival angles.

Index Terms—Fading channels, space-time codes, code diversity, long-range channel prediction, feedback delay, channel state information.

I. INTRODUCTION

In multiple-input multiple-output (MIMO) wireless communication systems, coding techniques such as space-time coding [1] were introduced to improve reliability of communication while maintaining a high transmission rate in an open-loop framework.

Many close-loop techniques were introduced to further improve the system performance. Beamforming (see [2–4]) is a general technique intended to improve channel capacity by feeding back the index for the best precoding matrix without considering specific code structures. Viswanath et al. [5] proposed opportunistic beamforming to optimize the total throughput for multi-user systems, where the base station induces channel fluctuation deterministically and selects the user with the highest SNR. Assuming the statistics of Channel State Information (CSI) are known at the transmitter, precoding technique can be used to minimize the average pair-wise error probability [4]. Yet none of these techniques take advantage of specific coding structures, and this work typically assumes instantaneous feedback. Code diversity has been proposed and intensively studied recently [6, 7] in the framework of space-time codes which impose structure on the transmitted signal. In the code diversity framework, a small number of feedback bits select the best code from a family of appropriately designed codes. Code diversity has been shown to greatly improve the decoding performance of sub-optimal low complexity decoders and complements the standard approach to code design by taking advantage of different (possibly equivalent) realizations of a particular code design. This paper will review the principle of code diversity, demonstrate its significance in terms of decoding performance/complexity, and discuss system implementation issues.

In previous work concerning code diversity, researchers assumed an ideal uncorrelated channel with a delay-free feedback link. In this paper, we consider time-varying correlated channels typical in practical mobile communication systems. We also suppose that there is a certain delay in the feedback link. In this setting, to accurately implement code diversity, we require long-range channel prediction [8, 9] to compensate for the feedback delay. We will review the linear autoregressive channel prediction algorithm and methods to increase its accuracy, especially in the low SNR regime. The design principles of combined code diversity and channel prediction will be developed, and simulations and comparisons will be provided to quantify the value of the combination. To test performance for correlated mobile radio channels, we employ two fading channel models: the conventional Jakes model and a realistic physical model.

The contributions of this paper are:

1) It is demonstrated that code diversity improves on Long Term Evolution (LTE) precoding by reducing the number of feedback bits.
2) It is shown that fading prediction enables code diversity in realistic mobile radio channels.
3) Finally, it is shown that the reliability of fading prediction is enhanced by noise reduction using oversampled pilots. Moreover, it is demonstrated that robustness to
prediction errors while maintaining low pilot rate is achieved by employing an outer encoder and an interleaver.

The message of the paper is that channel adaptivity, in this case the combination of code diversity and channel prediction, does result in significant performance gains in a realistic system.

The paper is organized as follows. Section II presents the system model and introduces the channel models. Code diversity is reviewed and compared to LTE precoding in Section III. Section IV describes the methodology of realizing code diversity through long-range channel prediction with high pilot rate and contains analysis and numerical results that illustrate the effect of prediction errors as well as simulations for the Jakes and physical models. In Section V, code diversity aided by low pilot rate prediction with interleaving is presented. The conclusion is provided in Section VI.

II. SYSTEM MODEL

A. Receive Structure

In a space-time coded wireless communication system with \(N_t\) transmit antennas and \(N_r\) receive antennas, the received signal \(Y\) is given by

\[
Y =HX + N \tag{1}
\]

where \(H \in \mathbb{C}^{N_r \times N_t}\) is the channel matrix with entry \(h_{ji}\) representing the Rayleigh fading channel gain between the \(i^{th}\) transmit antenna and the \(j^{th}\) receive antenna (\(h_{ji}\) are independent, identically distributed complex Gaussian variables); \(X\) represents a \(N_t \times T\) block codeword where \(T\) is the number of time slots one codeword covers, and \(N\) is the corresponding additive noise matrix. We suppose the additive noise is symmetric complex Gaussian with zero mean and variance \(2\sigma_n^2\). In this paper, we consider systems with multiple transmit antennas and a single receive antenna.

An important feature of many algebraic constructions of space-time codes is exchangeability of structure (correlation) at the transmitter and structure at the receiver. Exchangeability means that we can rewrite (1) as:

\[
y = \mathcal{H}(x + n) \tag{2}
\]

where \(y \in \mathbb{C}^{N_r \times T \times L}; \mathcal{H} \in \mathbb{C}^{N_r \times N_t \times T \times L}\); \(L\) is the number of distinct symbols transmitted during one block of \(T\) symbol periods; \(x \in \mathbb{C}^{L \times 1}\) is the transmitted signal, and \(n \in \mathbb{C}^{N_r \times T \times 1}\).

Equation (2) captures the perspective of the receiver, where the induced channel is assumed to be known, and the problem is to estimate the transmitted signals. Exchangeability of structure from the transmitter to the receiver is possible for (real-valued) linear dispersion codes [10], the Golden code [11–13], the Silver code [14] and many more.

B. Channel Model

We employ the Jakes fading model [15] to simulate the time-varying correlated Rayleigh fading from one frame to next. The Jakes model assumes that \(N\) equal-strength rays arrive at a moving receiver with uniformly distributed arrival angles \(\alpha_n = 2\pi n/N\), such that ray \(n\) experiences a Doppler shift \(\omega_n = 2\pi f_{dm} \cos(\alpha_n)\) where \(f_{dm}\) is the maximum Doppler shift in Hz.

To generate multiple independent waveforms, Dent at al. [16] proposed a construction using Walsh-Hadamard codewords. With \(N_0\) a power of two, the \(j^{th}\) waveform is generated as

\[
h_{ji}(t) = \frac{2}{N_0} \sum_{n=1}^{N_0} a_j(n) \exp(i\beta_n \cos(\omega_{ji}t+\theta_{ji})), j = 1 \cdots N_0 \tag{3}
\]

where \(\theta_{ji}\) and \(\beta_n\) are adjustable parameters, and \(A_j\) is the \(j^{th}\) Walsh-Hadamard codeword. This construction provides \(N_0\) uncorrelated channel gains used to model the channel coefficients for \(N_0\) transmit antennas.

We assume the channel is quasi-static, namely the channel remains constant during each frame of time. This assumption holds if the frame duration is \(\leq 1/(100f_{dm})\) [17]. For example, when \(f_{dm} = 100\text{Hz}\) and the data rate is 100Kbps, frame size \(\leq 10\) symbols would satisfy the quasi-static channel assumption. In our examples, \(T = 4\), so we assume the channel coefficients do not vary over the duration of one codeword.

Remark: While the Jakes model is usually employed to capture the characteristics of a correlated fading channel, it is an idealized model where the arrival rays have equal strengths and uniformly distributed arrival angles. In latter sections, we also employ a realistic physical model to test predicted code diversity [18].

III. CODE DIVERSITY

A. Algorithm and Analysis

Code diversity was introduced by Tan and Calderbank [6] and extensively studied in [7]. The code diversity scheme uses a small number of feedback bits to select the best code from a family of space-time codes. Later in the paper, we will review the phase adaptation method of inducing code diversity which is detailed in [7].

As illustrated in the system diagram of Fig. 1, in a phase adaptation framework, the receiver sends back feedback to modify phases of channel gains as follows

\[
h_{ji} \rightarrow h_{ji} \times e^{i\frac{\omega_{ji}f_{dm}}{K}}
\]

where \(k_{ij} \in \{0, 1, \ldots, K - 1\}\) is the feedback information and \(K\) determines the number of feedback bits. It is shown that the algorithm only needs to modify phases of a small subset of the channel gains.

Let \(\{h_{ij}\}\) denote the set where the phases of channel gains \(h_{ij}\) are modified and let \(\{k_{ij}\}\) denote the feedback selection information. Then the receiver selects \(\{k_{ij}\}\) which

- firstly maximizes the rank of \(\mathcal{H}^H\mathcal{H}\), i.e. \(d = \mathcal{R}(\mathcal{H}^H\mathcal{H})\) (Let \(d_{max} = \max \mathcal{R}(\mathcal{H}^H\mathcal{H})\))
- and secondly maximizes \((\prod_{k=1}^{d_{max}} \omega_i)\), where \(\omega_i\) are the nonzero eigenvalues of \(\mathcal{H}^H\mathcal{H}\).

If \(d_{max} = L\), i.e. \(\mathcal{H}^H\mathcal{H}\) has full rank, then the receiver selects \(\{k_{ij}\}\) as

\[
\{k_{ij}\} = \arg\max_{k_{ij} \in \{0, 1, \ldots, K - 1\}} \det(\mathcal{H}^H\mathcal{H})|_{\{h_{ij}\} \rightarrow \{h_{ij} \times e^{i\omega_{ji}}\}}. \tag{4}
\]
It is shown in [7] that given channel realization \( \mathcal{H} \), the average error probability \( P_{e|\mathcal{H}} \) is subject to
\[
P_{e|\mathcal{H}} \propto \left( \prod_{i=1}^{d} \omega_i \right)^{-1} \tag{5}
\]
and the channel capacity \( C_{\mathcal{H}} \) is subject to
\[
C_{\mathcal{H}} \propto \text{SNR}^d \log \left( \prod_{i=1}^{d} \omega_i \right). \tag{6}
\]
where SNR is the signal to noise ratio.

Therefore, by maximizing \( d \) and \( \left( \prod_{i=1}^{d} \omega_i \right) \), the code diversity scheme optimizes both the average error probability and the channel capacity in high SNR regime. In the code diversity framework, the performance of low complexity decoders can be significantly improved to match that of the maximum-likelihood decoders. The proposed code diversity scheme is in line with the design criteria (namely rank criterion and determinant criterion) for space-time coding [1] to maximize coding performance.

The system model is presented in Fig. 1. It employs space-time coding across multiple transmit antennas and optional convolutional coding and interleaving.

**B. Comparison of Code Diversity and LTE Precoding**

In this subsection, we compare the performance of code diversity and LTE precoding. The code diversity scheme can be applied to the family of exchangeable space-time codes. One example is Jafarkhani’s Quasi-Orthogonal Space Time Block Code (QOSTBC) [19], for which the encoding rule is given by
\[
X = \begin{pmatrix}
x_1 & x_2 & x_3 & x_4 \\
-x_2 & x_1 & -x_4 & x_3 \\
-x_3 & -x_4 & x_1 & x_2 \\
x_4 & -x_3 & -x_2 & x_1
\end{pmatrix} \tag{7}
\]
where four symbols \( x_1, x_2, x_3, x_4 \) are transmitted over four time slots via four transmit antennas. The induced channel matrix \( \mathcal{H} \) in equation (2) for QOSTBC is
\[
\mathcal{H} = \begin{pmatrix}
h_1 & h_2 & h_3 & h_4 \\
h_2 & h_1 & -h_4 & -h_3 \\
h_3 & -h_4 & h_1 & h_2 \\
h_4 & -h_3 & -h_2 & h_1
\end{pmatrix} \tag{8}
\]
where \( h_i \) is the channel coefficient from the \( i \)th transmit antenna to the single receive antenna. As shown in [7], we have
\[
\mathcal{H}^\dagger \mathcal{H} = \mathcal{H} \mathcal{H}^\dagger = \begin{pmatrix}
a & 0 & 0 & b \\
0 & a & -b & 0 \\
0 & -b & a & 0 \\
b & 0 & 0 & a
\end{pmatrix} \tag{9}
\]
where \( a = \sum_{i=1}^{4} |h_i|^2 \) and \( b = 2 \Re(h_1 h_4^* - h_2 h_3^*) \). Therefore,
\[
det(\mathcal{H}^\dagger \mathcal{H}) = (a^2 - b^2)^2. \tag{10}
\]

Given phase adaptation on the phase of \( h_1 \), the code diversity algorithm is
\[
k^* = \arg \max_{k \in \{0,1,...,K-1\}} \det(\mathcal{H}^\dagger \mathcal{H})|_{h_1 \rightarrow h_1 e^{j\pi k}}
\]
\[
= \arg \max_{k \in \{0,1,...,K-1\}} |a^2 - b^2|_{h_1 \rightarrow h_1 e^{j\pi k}} \tag{11}
\]
\[
= \arg \min_{k \in \{0,1,...,K-1\}} |b|_{h_1 \rightarrow h_1 e^{j\pi k}}
\]

There are totally 16 LTE precoders for systems with 4 transmit antennas with each one represented by
\[
P_i = I - 2 \frac{\mathbf{u}_i \mathbf{h}^H}{\mathbf{u}_i^H \mathbf{u}_i} \tag{12}
\]
where \( \mathbf{u}_i \) are listed in Table IV in the Appendix. Denote \( \mathbf{h} = (h_1, h_2, h_3, h_4)^T \) and \( \mathbf{h} \) as the channel coefficient vector with precoding,
\[
\tilde{\mathbf{h}} = P_i \mathbf{h}. \tag{13}
\]
Note that all \( \mathbf{u}_i \) are convertible by a unitary matrix. Namely,
\[
\mathbf{u}_i = \mathbf{U} \mathbf{u}_j \tag{14}
\]
with \( \mathbf{U} \) as a unitary matrix. Therefore, the precoding matrix can be represented as:
\[
P_i = I - 2 \frac{\mathbf{u}_i \mathbf{u}_i^H}{\mathbf{u}_i^H \mathbf{u}_i}
\]
\[
= I - 2 \frac{(\mathbf{U}_i \mathbf{I})(\mathbf{U}_i \mathbf{I})^H}{(\mathbf{U}_i \mathbf{I})^H(\mathbf{U}_i \mathbf{I})}
\]
\[
= \mathbf{U}_i \left( I - \frac{\mathbf{H}^H}{2} \right) \mathbf{U}_i^H \tag{15}
\]
with \( \mathbf{U}_i = \text{diag}(\mathbf{u}_i) \). From the above analysis,
\[
a = \tilde{\mathbf{h}}^H \tilde{\mathbf{h}} = \mathbf{h}^H P_i^H P_i \mathbf{h} = \mathbf{h}^H \mathbf{h} \tag{16}
\]
with \( \mathbf{h} = (h_1, h_2, h_3, h_4)^T \). Therefore, \( a \) is invariant under LTE precoding.

We also have
\[
b = \tilde{\mathbf{h}}^H \mathbf{B} \tilde{\mathbf{h}} = \mathbf{h}^H P_i^H \mathbf{B} P_i \mathbf{h} \tag{17}
\]
with
\[
\mathbf{B} = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0 \\
0 & -1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{pmatrix}. \tag{18}
\]
LTE precoding provides the following four options for the values of $b$:

\[
\begin{align*}
    b_1 &= h_1 h_4^* + h_2^* h_4 - h_2 h_4^* - h_3 h_2^*, \\
    b_2 &= h_1 h_3^* + h_3^* h_2 - h_3 h_2^* - h_4^* h_2, \\
    b_3 &= h_1 h_3^* + h_3^* h_3 - h_3 h_3^* - h_4 h_3^*, \\
    b_4 &= h_1 h_4^* + h_2 h_4^* - h_2 h_2^* - h_3 h_4^*.
\end{align*}
\]

These 16 LTE precoding matrices can be divided into four equivalent groups as in Table I. At this point, we have shown that 4 bits feedback information in LTE precoding reduces to only 2 bits. This reveals inefficiency of LTE precoding for QOSTBC and shows the importance of designing precoding matrices that are not connected by symmetries of the space-time code. Similar analysis can be drawn to show the inefficiency of LTE precoding for Diagonal Algebraic Space-Time (DAST) codes [20], circulant codes [21], etc.

The comparison of LTE precoding and code diversity is shown in Fig. 2. In the legend, ‘CD’ denotes code diversity. We note that the scheme of LTE precoding with 4 bits feedback has the same performance as code diversity with 2 bits feedback information. Code diversity with 3 bits feedback outperforms LTE precoding. We also note that the benefit of increasing feedback bits from 2 bits to 3 bits is negligible.

In the rest of this paper, QOSTBC is employed to illustrate different scenarios of code diversity with 2 feedback bits (total) aided by long-range fading prediction. Exhaustive search is employed to select the best code diversity system in equation (11).

### IV. Realizing Code Diversity through Long-Range Channel Prediction

The code diversity scheme considered in [6, 7] assumes that the feedback is instantaneous, i.e. there is no delay in the feedback link. In the mobile radio transmission environment, the channel is typically rapidly time-varying, and the channel state information (CSI) at the transmitter is outdated due to the feedback delay. To enable code diversity in mobile radio systems, we compensate for this delay by predicting the channel.

In this section, we illustrate integration of code diversity and long-range channel prediction by considering systems with only space-time coding. In the next section, we will illustrate further performance improvement by concatenating space-time coding with convolutional coding and interleaving.

#### A. Long-Range Channel Prediction

Long-range channel prediction was comprehensively studied in [8, 9]. It is based on the linear minimum mean square error (MMSE) principle.

Consider a Raleigh fading signal $h(t)$ sampled at the rate $f_n$. Suppose the current channel sample is $h_n$. The MMSE prediction of the future channel sample $h_{n+k}$, $k = 1, 2, \ldots$ based on $p$ previous samples $h_n, h_{n-1}, \ldots, h_{n-p+1}$ is given by

\[
    \hat{h}_{n+k} = \sum_{j=1}^{p} d_j h_{n+1-j}
\]

where $k/f_n$ is the prediction range (in seconds), $p$ is the predictor order and

\[
    d = [d_1, \ldots, d_p]^T = R^{-1}r
\]

where $r$ is the autocorrelation vector $(p \times 1)$ with coefficients $r_j = \mathbb{E}[h_{n+k} h_{n-j}^*]$ and $R$ is the $p \times p$ autocorrelation matrix with coefficients $R_{ij} = \mathbb{E}[h_{n-i} h_{n-j}^*]$. The correlation coefficient $R_{ij}$ and $r_j$ can be estimated from the observed channel samples without the prior knowledge of the maximum Doppler shift and the number or parameters of the scatterers. In practice, adaptive tracking methods are employed to keep up with the variation of the channel statistics [9].

The autocorrelation function of the Raleigh fading signal $h_n(t)$ is $r(\tau) = \mathbb{E}[h(t) h^*(t+\tau)]$ given by

\[
    r(\tau) = J_0(2\pi f_{dm}\tau)
\]

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind [15]. Given fixed filter size $p$, the predictor in equation (20) employs low sampling rate $f_n$ (5 to 10 times $f_{dm}$) and exploits large sidelobes of $r(\tau)$ in (21) to predict reliably sufficiently far ahead to enable adaptive transmission [8].

One method to obtain the estimates of the channel samples $h_n$ is to transmit pilot symbols at the low sampling rate $f_n$ employed in the predictor. Then the observed pilot symbols at the receiver are employed as channel samples. This method achieves good prediction accuracy at high SNR, but its performance is limited at medium and low SNR [22, 23]. To improve prediction accuracy, highly oversampled pilot symbols are employed to perform noise reduction prior to prediction in [22]. However, this method greatly reduces the...
spectral efficiency. To reduce the bandwidth occupied by pilots, a data-aided noise reduction method was proposed for adaptive modulation systems in [23, 24] and robust, low pilot rate prediction for adaptive bit-interleaved coded-modulation (ABICM) systems was investigated in [25]. We employ the adaptive modulation systems in [23, 24] and robust, low pilot rate prediction is extended to code diversity systems in the high pilot rate method in this section. Efficient low pilot rate systems was investigated in [25]. We employ the ABICM systems in [25] and filter order 50. The outputs of these noise-reduction filters are down-sampled at the rate of 10f_{\text{sam}} and then fed into the MMSE predictor, which has the filter order of 20. We also show results for the low pilot rate method that does not employ noise reduction. The parameters of the linear MMSE predictor are summarized in Table II.

In our simulations, the high pilot rate method employs pilot symbols transmitted at the rate of 100f_{\text{sam}}. The noise reduction is performed using four filters with smoothing lags \{0, 2, 5, 10\} [23] and filter order 50. The outputs of these noise-reduction filters are down-sampled at the rate of 10f_{\text{sam}} and then fed into the MMSE predictor, which has the filter order of 20. We also show results for the low pilot rate method that does not employ noise reduction. The parameters of the linear MMSE predictor are summarized in Table II.

Fig. 3 illustrates actual and predicted channel gains for four independent channels.

B. Performance analysis of predicted CD

Assuming zero-delay and error-free channel feedback, the code diversity algorithm is given by (11). In practice, the channel feedback is usually delayed. By employing prediction of coefficient \( h_i \) as described in section IV-A, we obtain the predicted induced channel matrix \( \hat{H} \) given by:

\[
\hat{H} = \begin{pmatrix}
\hat{h}_1 & \hat{h}_2 & \hat{h}_3 & \hat{h}_4 \\
-h_2^* & \hat{h}_1^* & -h_3^* & \hat{h}_4^* \\
-h_3^* & -h_4^* & \hat{h}_1^* & \hat{h}_2^* \\
\hat{h}_4^* & -\hat{h}_3^* & -\hat{h}_2^* & \hat{h}_1^*
\end{pmatrix},
\]

where \( \hat{h}_i \) are predicted channel coefficients. The resulting predicted code diversity algorithm decides on

\[
\hat{k}^* = \arg \max_{k \in \{0, 1, \ldots, K-1\}} \det(\hat{H}^\dagger_1) \big|_{h_i \to \hat{h}_i e^{i \Delta}}
\]

\[
= \arg \min_{k \in \{0, 1, \ldots, K-1\}} |\hat{b}| \left|_{h_i \to \hat{h}_i e^{i \Delta}}
\]

where \( \hat{b} = 2\Re(\hat{h}_1^* \hat{h}_4 - \hat{h}_2^* \hat{h}_3^*) \).

The quality of MMSE channel prediction (20) or mismatched CSI is characterized by its cross-correlation with channel coefficient

\[
\rho = \frac{E[\{h(t)\hat{h}^*(t)\}]}{\sqrt{E[\{h(t)^2\}]E[\{\hat{h}(t)^2\}]}}
\]

Substituting equation (10) in equation (5), the decoding error probability of QOSTBC can be expressed as

\[
P_{e|h} \propto \frac{1}{\det(\hat{H}^\dagger_1)} \propto (1 - \lambda^2)^{-2} a^{-4},
\]

where \( \lambda = |\frac{h}{a}|, a = \sum_{i=1}^{4} |h_i|^2 \) and \( b = 2\Re(h_1 h_4^* - h_2 h_3^*) \).

We analyze the impact of prediction errors on the parameters in (25) below.

First, for the purpose of analysis, we assume \(|h_i| = 1\). Then \( \lambda \) can be written as:

\[
\lambda_0 = \frac{\cos \theta_1 - \cos \theta_2}{2}
\]

where \( \theta_1, \theta_2 \in [0, \pi] \) are the phases of \( h_1 h_4^* \) and \( h_2 h_3^* \), respectively. We can further write \( \theta_2 = \theta_1 + \psi \) with \( \psi \in [0, \pi] \).

Code diversity scheme minimizes \( \lambda \) by adapting channel phases while keeping \( \rho \) unchanged. When there is no prediction error, i.e. when \( \rho = 1 \), \( \lambda \) can be forced to 0 by inducing phase adaptation on \( h_1 \) and \( h_2 \) with a sufficient number of feedback bits. In this case, the performance of code diversity scheme is optimized.

When \(|\rho| < 1\), the performance of code diversity based on the predicted channel coefficients is impaired by prediction errors. Assuming a phase shift \( \epsilon \), incurred by the prediction error on \([h_1, h_2, h_3, h_4]\), the induced \( \lambda \) can be written as

\[
\hat{\lambda} = \frac{|\cos \theta_1 - \cos (\theta_1 + \epsilon)|}{2}
\]

where \( \epsilon \) is determined by \( \rho \). It is now clear that if \( \epsilon \) is large enough so that \( \mathbb{E}\theta_1 \hat{\lambda} \leq \mathbb{E}(\theta_1, \psi) \lambda_0 \), code diversity does not help. Assuming \( \theta_1, \psi \) follow uniform distribution on \([0, \pi] \), we obtain the boundary condition of conducting code diversity via simulation:

\[
\epsilon \leq \frac{9}{25} \pi.
\]
complex channel coefficient vector $\hat{h}$ when prediction errors are significant. For given value of $|\rho|$, $E(\hat{h}^2)$ is demonstrated in Fig. 4 that resulting expectation in (29) are obtained by simulation. It is not the actual value of $b$ but the complex, the correlated Gaussian random variables and the variables $h_i$ and $\hat{h}_i$ are independent for $i \neq j$ and the cross-correlation of $h_i$ and $\hat{h}_i$ is given by $\rho$. Then the expectation of $b^2$ can be expressed as

$$E(b^2) = \int_{\mathbb{C}^2} b^2 p(h, \hat{h}) dh d\hat{h}.$$ \hspace{1cm} (29)

In (29), an eight-dimensional integration is required. Since numerical calculation of this equation is computationally complex, the correlated Gaussian random variables and the resulting expectation in (29) are obtained by simulation. It is demonstrated in Fig. 4 that $E(b^2)$ approaches the “no CD” level as $\rho$ approaches zero. The value of $E(b^2)$ saturates for $|\rho| < 0.5$ As $\rho$ approaches 1, the value of $E(b^2)$ decreases to the ideal CSI case.

We will relate these insights to the BER of CD aided by fading prediction in section IV-C.

C. Predicted Code Diversity for Jakes Model

In this subsection, we use Jafarkhani’s QOSTBC to illustrate the integration of code diversity and fading prediction. In a $4 \times 1$ system, the induced channel matrix $\hat{H}$ in equation (2) for QOSTBC is given by (8). The channel coefficients $h_1$, $h_2$, $h_3$ and $h_4$ are generated from the modified Jakes model in equation (3).

Simulations in Fig. 5 and Fig. 6 show performance of combined CD and fading prediction for the linear ZF decoding and ML decoding with symbols on 4-QAM. In these figures, ‘Without CD’ represents decoding without employing code diversity; ‘Mismatch-CD’ represents performance of code diversity using outdated CSI, i.e. prediction is not employed; ‘Pred-CD’ and ‘LR-Pred-CD’ represent CD aided by high pilot rate and low pilot rate long-range channel prediction, respectively; ‘CD’ represents code diversity with perfect CSI. It is shown that the performance of code diversity aided by the high pilot rate predictor approaches that of the perfect code diversity scheme while using the low pilot rate predictor or outdated CSI results in significant performance degradation (over 1.5 dB and 3 dB, respectively) and diversity loss.

Now we vary the prediction range for the linear ZF decoding of Jafarkhani’s QOSTBC. The high pilot rate method is employed. Simulations in Fig. 7 show that as the prediction range increases, the performance of the predicted code diversity scheme deteriorates to the performance of the scheme without code diversity. The result can be related to the adaptation performance in Fig. 4 by noting that the cross correlation (24) $|\rho| > 0.95$ when the prediction range is 1ms, but decreases to $|\rho| \approx 0.65$ and 0.7 for $SNR = 9$ and 12dB, respectively, when the prediction range is 5ms. Finally, for 10ms prediction range, $|\rho| \approx 0.55$ for both SNR values. Thus, $b^2$ grows and the benefit of adaptation diminishes rapidly with prediction range.
In practice, prediction ranges of 2-3ms are often sufficient to compensate for the feedback delay. For the latter ranges, $|\rho| \geq 0.85$, and prediction is sufficiently accurate to enable CD.

D. Predicted Code Diversity in a Physical Environment

While the Jakes channel model is conventionally used to simulate a time-varying correlated mobile radio channel, it does not capture the environmental properties that impact performance of long-range fading prediction. We have developed a physical channel model based on the method of images and augmented with Fresnel diffraction and have demonstrated that it generates datasets with time-variant statistical characterization and results of the prediction performance similar to those for measured data and differing significantly from those for the Jakes model data [8, 18]. In this subsection, we employ a realistic physical model dataset for the geometry shown in Fig. 8. This data set represents a realistic scenario where the incoming angles of reflections vary fast due to the reflectors closely placed to the route of mobile user. The nonstationary nature of this data set makes it challenging to predict [18]. To keep up with channel parameter variations, we employ the Burg predictor with high pilot rate-based noise reduction proposed in [27].

In this scenario, the antenna array is installed at the base-station while the mobile user has only one antenna. Table III shows additional parameters used in the simulation. The predictor parameters and the Doppler frequency are the same as in Table II.

Simulation in Fig. 9 shows that the high pilot rate predicted code diversity scheme matches the BER of the perfect code diversity scheme and significantly outperforms the ‘Mismatch-CD’, ‘LR-Pred-CD’ and ‘without CD’ schemes in realistic channel settings by 1.5-5dB. We observe that the gain due to prediction is similar to that in the Jakes model scenario although the delay (prediction range) is increased from 2 ms to 3 ms. This can be explained by a lower number and a nonuniform distribution of the reflectors that mitigates the time variation of fading in the physical model data set [8, 18, 23, 24].

V. Realizing Code Diversity Using Robust, Low Pilot Rate Prediction

A. Spectral Efficiency and Pilot Rate

In previous simulations, we employ pilots transmitted at a much higher rate than the maximum Doppler frequency and than the rate required for long range prediction in high SNR [8]. These highly oversampled pilots are necessary to reduce noise and thereby improve the accuracy of channel estimation. The noise-reduced channel estimates are then sampled at a lower rate to generate improved long-range prediction results. However, this approach also consumes considerable channel bandwidth, limiting the overall spectral efficiency, as was demonstrated for adaptive modulation systems in [24].

To improve the spectral efficiency, two approaches have been proposed and demonstrated for adaptive modulation systems [23–25, 28]. The first method is to maintain the prediction accuracy using data-aided noise reduction [23, 24]. In this method, the pilot symbols are transmitted at a low rate (5-10 times the $f_{dm}$) necessary for fading prediction (20), and the decisions of data symbols are employed to improve channel estimation accuracy prior to prediction.

Another approach is to enhance the robustness of adaptive transmission to imperfect channel prediction. In [25, 28], it
was shown that the adaptive bit-interleaved coded-modulation (ABICM) method significantly improves upon the adaptive modulation techniques that do not employ interleaving when the channel estimation is unreliable. In ABICM, the data bits associated with poor predictions are spread over a long time interval to improve the decoding accuracy. This approach is extended to improve the robustness of prediction-enabled code diversity below.

In some practical systems such as WiMAX and LTE, a large number of pilots are transmitted to obtain channel estimation. For those systems, it is possible to use highly oversampled pilots to reduce noise without causing additional spectral efficiency penalty. For example, in WiMAX, the pilot symbols are transmitted at about 9.7K samples/s [29], which is sufficient for noise reduction if \( f_{\text{dm}} < 100\,\text{Hz} \). We illustrate the combined benefit of noise reduction and interleaving below.

### B. Channel Prediction with Low Pilot Rate

In MIMO communications, encoders typically consist of two layers. One layer is the channel coding, such as convolutional codes or LDPC codes. The other layer is the space-time coding that schedules streaming and cooperation across multiple transmit antennas. An interleaver is usually placed between these layers to mitigate the error bursts caused by deep fades. In addition, interleaving enhances robustness to prediction errors and facilitates low pilot rate prediction in the low-to-medium SNR regime [25].

Consider the system in Fig. 1 that employs a 2/3-rate convolutional code illustrated in Fig. 10 as the outer code and QOSTBC as the space-time code. We assume an ideal interleaver as in [28]. In practical systems, this assumption is realistic for short interleaving depth in time due to the presence of frequency and/or space diversity. In the low pilot rate method, we transmit the pilot symbols at the rate of \( 10\, f_{\text{dm}} \). Thus, the bandwidth occupied by the pilot symbols is greatly reduced compared with the oversampled method described in Section IV. The observed pilot samples at the receiver are fed to the predictor (20) directly without noise reduction. The order of the MMSE predictor is 20.

We illustrate the performance of code diversity enabled by low pilot rate channel prediction for linear ZF decoding of QOSTBC for the Jakes and physical models in Fig. 11 and Fig. 12, respectively. In the legend, ‘-NI’ denotes ‘without interleaving’, ‘-I’ denotes ‘with interleaving’, and perfect CSI is assumed in CD-I and CD-NI curves. We also show the BER of the high pilot rate method with interleaving, denoted by “HR-Pread-CD-I”. Simulation parameters are specified in Tables II and III.

As expected, interleaving significantly improves performance since it distributes the error bursts caused by deep fading. In addition, interleaving is necessary to facilitate bandwidth-efficient low pilot rate prediction. Without interleaving, low pilot rate prediction is not accurate in the low and medium SNR regime as illustrated in Fig. 5, 6, 9 and [23, 24]. However, predicted CD aided by the low pilot rate predictor has reliable performance when interleaving is employed since the symbols associated with poor predictions are spread over a long time interval. Hence, interleaving reduces the sensitivity to the prediction errors and is within 1dB of the perfect CSI case. While interleaving improves robustness to CSI mismatch, prediction is still necessary to maintain low bit error rate, since predicted CD significantly outperforms the mismatched CSI method when interleaving is employed. Finally, the combination of interleaving and noise reduction-aided fading prediction (the high pilot rate method) approached the ideal CSI performance.

### VI. Conclusion

It was demonstrated that code diversity improves the efficiency of suboptimal low complexity decoders. In particular, using only two feedback bits, it matches the BER of LTE decoding with four feedback bits.

It was also shown that fading prediction enables code diversity for rapidly time-varying, fading channels. To reduce the effects of noise, a fading predictor aided by a noise reduction method that employs high-rate pilots and a robust, bandwidth efficient, low pilot rate predictor scheme with interleaving were employed. The performance is validated using the conventional Jakes model and a realistic physical channel model.

The low pilot rate method relies on an outer encoder and interleaver to preserve prediction accuracy at low SNR. It operates at the pilot rate of \( 10\, f_{\text{dm}} \). The high pilot rate

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**Fig. 9.** Linear ZF decoding of QOSTBC using predicted code diversity in a physical model.

**Fig. 10.** The diagram of the 2/3 rate convolutional code.
The gain provided by prediction is lower for a realistic physical model data set where fading signal variation is less severe than for the Jakes model data.

**APPENDIX**

**REFERENCES**


[23] T. Jia, A. Duel-Hallen, and H. Hallen, “Performance of adaptive coded modulation enabled by long-range fading prediction with data-aided noise reduction,” in *Proc. 2008 IEEE International Conference for Military Communications*. [![Fig. 11. Linear ZF decoding of QOSTBC using code diversity realized through low pilot rate prediction.](image1)](image1) [![Fig. 12. Linear ZF decoding of QOSTBC using code diversity realized through low pilot rate prediction for a physical model.](image2)](image2)

**TABLE IV**

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**Fig. 11.** Linear ZF decoding of QOSTBC using code diversity realized through low pilot rate prediction.

**Fig. 12.** Linear ZF decoding of QOSTBC using code diversity realized through low pilot rate prediction for a physical model.


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Dr. Calderbank served as Editor in Chief of the IEEE TRANSACTIONS ON INFORMATION THEORY from 1995 to 1998, and as Associate Editor for Coding Techniques from 1986 to 1989. He was a member of the Board of Governors of the IEEE Information Theory Society from 1991 to 1996 and began a second term in 2006. Dr. Calderbank was honored by the IEEE Information Theory Prize Paper Award in 1995 for his work on the $Z_4$ linearity of Kerdock and Preparata Codes (joint with A. R. Hammons Jr., P. V. Kumar, N. J. A. Sloane, and P. Sole), and again in 1999 for the invention of space-time codes (joint with V. Tarokh and N. Seshadri). He received the 2006 IEEE Donald G. Fink Prize Paper Award and the IEEE Millennium Medal, and was elected to the US National Academy of Engineering in 2005.

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