Performance of Improved Channel Allocation for Multicarrier CDMA with Adaptive Frequency Hopping and Multiuser Detection

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Abstract—Subchannel allocation for the reverse link of multicarrier code-division multiplex-access (MC-CDMA) system with adaptive frequency hopping (AFH) is investigated. An efficient allocation method that can be employed with the matched filter (MF) receiver as well as with multiuser detectors (MUDs) by exploiting the appropriate signal to interference and noise ratio (SINR) analysis is described. It is demonstrated that combined improved subcarrier allocation and the linear decorrelating detector (LDD) are very efficient in mitigating multiple-access interference (MAI), resulting in much larger system capacity than for a non-adaptive MC direct-sequence (DS)-CDMA system. This conclusion is confirmed for realistic fading channels with correlated subcarriers and imperfect channel state information (CSI) at the transmitter.

I. INTRODUCTION

MC-CDMA with AFH was addressed for the forward link in [1]–[3] and for the reverse link in [4], [5]. This adaptive transmission method exploits both frequency and multiuser diversity and improves on non-adaptive MC DS-CDMA systems in [6]. In this paper, we focus on the reverse link system, where multiple substreams are employed for each user. A sub-optimal water-filling (WF) allocation algorithm was proposed for the reverse link in [4]. It was discussed in [5] that the WF algorithm offers limited protection to weaker users and therefore suffers from the near-far problem caused by short-term fading. An improved MC-CDMA system with AFH that exploits quasi-synchronous reverse link model [7] was investigated, and an improved allocation algorithm was proposed and demonstrated to overcome the limitations of the WF algorithm [5].

Non-realistic assumptions of perfect knowledge of the CSI for the allocation algorithm and independent fading for all subcarriers were employed in [4], [5]. The impact of noisy CSI measurement was considered in [1]–[3]. However, for the reverse link, imperfect CSI is mainly caused by the feedback delay associated with sending channel allocation instructions from the base-station to mobile users in a rapidly varying fading channel. In addition, it is well known that correlated subcarriers reduce the frequency diversity of MC systems [8], and the assumption of independent subcarriers in [1], [4]–[6] tends to overestimate the system performance. In this paper, the impact of imperfect CSI and correlated subcarriers is investigated.

In [4], the conventional MF receiver, which is limited by MAI for a large number of users, was employed. It was shown that allocation algorithms, including the WF algorithm and the improved algorithm in [5], can be employed jointly with the LDD by utilizing SINR analysis [5], [9]. In this paper, we also utilize the improved allocation algorithm with the successive interference canceller (SIC), and compare its performance for several detectors.

The remainder of this paper is organized as follows. In section II, we describe the reverse link model of the improved MC-CDMA system with AFH. The SINR analysis for several receivers and the allocation algorithm in [5] are summarized, and the impact of imperfect CSI is discussed in section III. Finally, numerical results and conclusion are contained in Sections IV and V, respectively.

TABLE I: Notation

<table>
<thead>
<tr>
<th>symbol</th>
<th>description</th>
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<tbody>
<tr>
<td>$K$</td>
<td>number of users</td>
</tr>
<tr>
<td>$M$</td>
<td>number of subcarriers</td>
</tr>
<tr>
<td>$N$</td>
<td>number of substreams/user</td>
</tr>
<tr>
<td>$k, k'$</td>
<td>user indices</td>
</tr>
<tr>
<td>$m, m'$</td>
<td>subcarrier indices</td>
</tr>
<tr>
<td>$n, n'$</td>
<td>substream indices for a user</td>
</tr>
<tr>
<td>$p, p'$</td>
<td>substream indices on a given subcarrier</td>
</tr>
<tr>
<td>$(k, n)$</td>
<td>the $n$th substream of the $k$th user</td>
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<tr>
<td>$E_b$</td>
<td>bit energy</td>
</tr>
<tr>
<td>$N_0$</td>
<td>power spectral density of channel Gaussian noise</td>
</tr>
<tr>
<td>$T_c$</td>
<td>chip duration</td>
</tr>
<tr>
<td>$T_b$</td>
<td>bit duration</td>
</tr>
<tr>
<td>$PG$</td>
<td>processing gain = $t_b/T_c$</td>
</tr>
<tr>
<td>$\Delta f$</td>
<td>subcarrier bandwidth</td>
</tr>
<tr>
<td>$\sigma_d$</td>
<td>rms delay spread</td>
</tr>
<tr>
<td>$q_{k,n}$</td>
<td>allocation variable for substream $(k, n)$</td>
</tr>
<tr>
<td>$\gamma_{k,m}(t)$</td>
<td>fading coefficient on the $m$th subcarrier of user $k$</td>
</tr>
<tr>
<td>$\hat{\gamma}_{k,m}(t)$</td>
<td>prediction of $\gamma_{k,m}(t)$</td>
</tr>
<tr>
<td>$\alpha_{k,m}(t)$</td>
<td>amplitudes of $\gamma_{k,m}(t)$, $\hat{\gamma}_{k,m}(t)$</td>
</tr>
<tr>
<td>$\phi_{k,m}(t)$</td>
<td>phase of $\gamma_{k,m}(t)$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>cross-correlation between $\gamma_{k,m}(t)$ and $\hat{\gamma}_{k,m}(t)$</td>
</tr>
<tr>
<td>$U_m$</td>
<td>set of all substreams assigned to subcarrier $m$</td>
</tr>
<tr>
<td>$P_m$</td>
<td>number of substreams in subcarrier $m$</td>
</tr>
<tr>
<td>$\lambda(m,p)$</td>
<td>SINR for the $p$th substream on subcarrier $m$</td>
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</table>

II. SYSTEM MODEL

A. Transmitted Signal

System diagram of MC-CDMA with AFH is illustrated in Fig. 1. In the system with $K$ users, the total bandwidth $W$ is divided into $M$ subcarriers with equal bandwidths $\Delta f = W/M$. The data stream of each user is multiplexed
B. Channel Model and Imperfect CSI

We assume that the signal transmitted over each subcarrier experiences slowly varying flat Rayleigh fading [10], and the fading coefficients for different users are independent and identically distributed (i.i.d.). In addition, the equivalent low-pass channel coefficients $\gamma_{k,m}(t)$ for all subcarriers of the same user are identically distributed.

To assess realistic system performance, we employ correlated subcarriers in the system model. Assuming the propagation delay is exponentially distributed with rms delay spread $\sigma_d$ [11, pp. 50], the cross-correlation between the fading coefficients of two subcarriers of user $k$ at time $t$ with index difference $\Delta m = m - m'$ is given by [12]

$$r_f(\Delta m \Delta f) \triangleq \mathbb{E}\left[\gamma_{k,m}(t)\gamma_{k,m'}^*(t)\right] = \frac{1 + 2j\pi \Delta m \Delta f \sigma_d}{1 + (2\pi \Delta m \Delta f \sigma_d)^2}$$

(3)

In the reverse link, the base-station accurately estimates the current CSI of all users at each subcarrier by employing pilot signals [13]. However, due to the delay caused by the calculation of allocation variables and feeding the allocation assignments back to mobile users, the CSI used for allocation is different from the actual channel conditions experienced during the transmission. An efficient approach to reliable transmission in the presence of CSI mismatch is to predict future CSI $\gamma_{k,m}(t)$ based on outdated observations using the autoregressive (AR) model based linear prediction (LP) $\hat{\gamma}_{k,m}(t)$ [14], [15]. The mean square error (MSE) of this predictor is calculated as $\sigma^2_\Delta \triangleq \mathbb{E}\left[\gamma_{k,m}(t) - \hat{\gamma}_{k,m}(t)\right]^2$. The random variables $\hat{\gamma}_{k,m}(t)$ and $\gamma_{k,m}(t)$ are jointly Gaussian with the subcarrier-independent cross-correlation $\eta \triangleq \mathbb{E}\left[\gamma_{k,m}(t)\gamma_{k,m}^*(t)\right] / \sqrt{\mathbb{E}\left[\gamma_{k,m}(t)^2\right]\mathbb{E}\left[\gamma_{k,m}^*(t)^2\right]}$ and $\sigma^2_\eta = 1 - \eta^2$ [16]. For example, $\eta = 1$ implies perfect CSI, while over-sampled pilot-aided LP with noise reduction achieves $\eta=0.95$ at SNR=20dB for predicting 0.3 wavelengths ahead in space [17]. Implementation of reliable prediction algorithm for this application is outside the scope of this paper. To evaluate performance for the prediction accuracy specified by the reliability parameter $\eta$ in simulations, we generate predicted channel coefficients as $\hat{\gamma}_{k,m}(t) = \eta^2 \gamma_{k,m}(t) + \omega(t)$, where $\omega(t)$ is white Gaussian noise with variance $\eta^2 (1 - \eta^2)$ that is independent of $\gamma_{k,m}(t)$.

III. RECEIVER DESIGN AND THE ALLOCATION ALGORITHM

A. Receiver Design and SINR Analysis

The receiver structure of the proposed system was shown in [5, Fig. 2]. We assume that the inter-subcarrier, inter-symbol, and inter-user interference between adjacent symbols is negligible and timing and frequency synchronization are perfect. As discussed in [5], [9], numerical results obtained under these assumptions closely approximate realistic system performance for the quasi-synchronous channel.

After the received signal is passed through a chip-matched filter, followed by the chip-rate sampler, the chip-rate samples are correlated with the local spreading sequence references $c_{k,n}$ [4], [5]. Without loss of generality, we consider only
one output symbol of this correlator and use the slow fading assumption to denote $\gamma_{k,m} \triangleq \gamma_{k,m}(t)$, $\phi_{k,m} \triangleq \phi_{k,m}(t)$, and $\tilde{\varphi}_{k,m} \triangleq \tilde{\varphi}_{k,m}(t)$ during this symbol period.

Define $U_m$ as the index set of all substreams that are allocated to the $m$th subcarrier, i.e., $U_m \triangleq \{(k,n) | q_{k,n} = m\}$. Suppose the substream $(k,n) \in U_m$. The output of the correlator for this substream is given by [4]–[6]

$$Z_k,n = \tilde{S}_{k,n} + \tilde{I}_{k,n} + \tilde{N}_{k,n}$$  \hfill (4)

where $\tilde{S}_{k,n} = \sqrt{E_b}\alpha_{k,m} b_{k,n}$ carries the desired bit information, and $\tilde{I}_{k,n}$ is the interference term

$$\tilde{I}_{k,n} = \sqrt{E_b} \sum_{(k',n') \in U_m} \alpha_{k',m} b_{k',n'} \rho(k',n')(k,n)$$  \hfill (5)

The cross-correlation $\rho(k',n')(k,n)$ between the waveforms of corresponding substreams is calculated as [9], [18]

$$\rho(k',n')(k,n) \triangleq \frac{T_s}{T_c} \left[ \sum_{i=0}^{PG-1} c_{k',n'}(i) h(t - i T_c) \right]$$ \hfill (6)

The covariance between the zero-mean Gaussian noise terms of $\tilde{N}_{k,n}$ in (4) is $\mathbb{E}\left[\tilde{N}_{k,n} \tilde{N}_{k',n'}^{*}\right] = N_0 \rho(k',n')(k,n)$ for all $q_{k,n} = q_{k',n'}$. When the number of users $K$ is large, the interference $\tilde{I}_{k,n}$ can be modeled as zero-mean Gaussian noise with the variance [4]–[6]

$$\text{Var}(\tilde{I}_{k,n}) = \frac{E_b}{PG} \sum_{(k',n') \in U_m, k' \neq k} \alpha_{k',m}^2$$  \hfill (7)

Consider the outputs of the MFs for $P_m$ substreams allocated to the $m$th subcarrier. Without loss of generality, replace the substream index pair $(k,n)$ in (4) with a single index $p$, i.e.,

$$Z_p = \tilde{S}_p + \tilde{I}_p + \tilde{N}_p, p \in [1, \ldots, P_m]$$  \hfill (8)

and define the indicator function

$$\Delta(p',p) \triangleq \Delta[(k',n'),(k,n)] \triangleq \begin{cases} 1, & \text{if } k' \neq k, \\ 0, & \text{otherwise} \end{cases}$$  \hfill (9)

Let the channel amplitude associated with substream $p$ be $\alpha_p$. For the MF receiver, the SINR of the $p$th substream can be approximated as [4], [5]

$$\lambda_{MF}(m,p) \approx \frac{E_b\alpha_p^2}{\frac{E_b}{PG} \sum_{p'=1}^{P_m} \Delta(p',p)\alpha_{p'}^2 + N_0}$$  \hfill (10)

To reduce MAI, we apply MUD at the MF output (8). First, consider the LDD. In the matrix form, the output of the MF (8) can be rewritten as $z = RAb + \hat{n}$ [18, pp. 234], where $z = [Z_1 Z_2 \ldots Z_{P_m}]^T$, $P_m \times P_m$ cross-correlation matrix $R$ has components $R_{pp'} = p p'$, $A = \text{diag} \{\sqrt{E_b}\alpha_1, \sqrt{E_b}\alpha_2, \ldots, \sqrt{E_b}\alpha_{P_m}\}$, $b = [b_1, b_2, \ldots, b_{P_m}]^T$, and $\hat{n} = [\hat{N}_1, \ldots, \hat{N}_{P_m}]^T$. The covariance matrix of $\hat{n}$ is

$N_0 R$. The output of the LDD is given by $R^{-1}z = Ab + R^{-1}\hat{n}$, and the SINR of the LDD for the $p$th substream of the $m$th subcarrier is [18, pp. 249]

$$\lambda_{LDD}(m,p) = \frac{E_b\alpha_p^2}{\left[ N_0 \left( R^{-1} \right)_{p,p} \right]}$$  \hfill (11)

When the SIC is applied to (8), we assume without loss of generality that $\alpha_1 \geq \alpha_2 \geq \ldots \geq \alpha_{P_m}$. In the $p$th stage, the remaining user with the strongest received signal is demodulated as

$$\hat{b}_p = \text{sgn}\left( Z_p - \sum_{p'=1}^{p-1} \sqrt{E_b}\alpha_{p'} \hat{b}_{p'} \right)$$  \hfill (12)

where $\text{sgn}(\cdot)$ is the sign function. The SINR of the $p$th substream on the $m$th subcarrier can be approximated as [18, pp. 360]

$$\lambda_{SIC}(m,p) \approx \frac{E_b\alpha_p^2}{\left[ N_0 + \frac{E_b}{PG} \sum_{p'=1}^{P_m} \Delta(p',p)\alpha_{p'}^2 \right]}$$ \hfill (13)

where the bit-error rate (BER) of the $p$th substream is $P_{e,SIC}(m,p') \approx Q\left[\sqrt{2\lambda_{SIC}(m,p')}\right]$, $Q(x) = \int_x^\infty \exp(-t^2/2)dt/\sqrt{2\pi}$, and $\Delta(p',p)$ is defined in (9).

**TABLE II: Improved Allocation Algorithm**

Initialize $P_m = 0$ and $U_m = \{\emptyset\}$ for all $m \in [1, \ldots, M]$.

for $n = 1$ to $N$

for $k$ from 1 to $K$

S1) Augment $U'_m = \{U_m, (k,n)\}$ for $m \in [1, \ldots, M]$.

Let $\lambda(m,p)$ be the SINR of the $p$th substream ($p \in [1, \ldots, P_m + 1]$) assuming that substream $(k,n)$ is assigned to the $m$th subcarrier.

S2) Find $m_o$ that satisfies

$$m_o = \arg\max_{m \in [1, \ldots, M]} \left\{ \min_{p \in [1, \ldots, P_m + 1]} \lambda(m,p) \right\}$$

S3) Assign substream $(k,n)$ to the $m_o$th subcarrier, i.e., set $q_{k,n} = m_o$. Then update $P_{m_o} = P_{m_o} + 1$, and $U_{m_o} = U'_{m_o}$.

**B. Improved Channel Allocation Algorithm**

In the reverse link of the MC-CDMA system with AFH, the allocation algorithm is implemented at the base-station, and the resulting allocation assignments $\{q_{k,n}\}$ are sent back to the mobiles, where they are employed to select the reverse link subcarriers for each substream. Thus, the computational load associated with channel allocation is borne entirely by the base-station that can afford much greater complexity than the mobiles. Moreover, the feedback load is low since only a few allocation bits need to be fed back at a low rate that is on the order of the maximum Doppler shift.
The limitations of the WF algorithm investigated in [4] were discussed in [5]. Moreover, a novel allocation method shown in Table II that improves upon the WF method was proposed in [5]. When perfect CSI is available at the base-station, the SINR in step S1 of this improved allocation algorithm is calculated using the expression (10), (11) or (13), depending on the type of the detector employed at the receiver. As discussed earlier, this perfect knowledge of CSI assumed in the calculation of \( \eta \) is usually unavailable. We employ the predicted fading amplitude \( \hat{\alpha}_p \) to replace \( \alpha_p \) in the SINR \( \lambda(m,p) \), and obtain the SINR estimate \( \hat{\lambda}(m,p) \) that is used as performance measure in step S1 of our allocation algorithm.

The complexity of the proposed allocation algorithm with the LDD at the receiver can be significantly reduced by using iterative matrix inversion method as discussed in [5]. Moreover, assuming \( KN \gg M \), the allocation algorithms for the LDD and the SIC have the same order of complexity \( O(K^3N^3) \), while the allocation algorithm for the MF has complexity on the order of \( O(K^2N) \) [5], [9].

IV. NUMERICAL RESULTS

It has been demonstrated in [5] that the improved allocation method outperforms the WF algorithm, and that significant gain is achieved relative to non-adaptive MC DS-CDMA system when the CSI knowledge is perfect. Fig. 2 shows that for \( \eta = 1.0 \) the BER of MC-CDMA system with AFH and LDD approaches the single-user bound (SUB) of the same system for medium system load (K is comparable to M), while the BER of SIC is much higher for medium and high SNR. In moderately loaded adaptive MC-CDMA system, the LDD does not suffer from noise enhancement since the allocation algorithm separates the substreams with high cross-correlations into different subcarriers. In addition to inferior performance of SIC and MF detectors relative to the LDD [18], the allocation method employed with the former detectors is impaired by the inaccuracy of the SINR approximations (10) and (13) for small number of substreams, as in the first stages of the allocation method. We also investigate the effect of imperfect CSI in Fig. 2. Observe that while the BER of the adaptive system with LDD increases as the reliability of CSI degrades, it is still smaller than the SUB of non-adaptive MC DS-CDMA even for \( \eta = 0.95 \), which is easily achievable in realistic scenarios [15], [17].

Fig. 3 illustrates the impact of correlated subcarriers on the BER of the adaptive system when perfect knowledge of the CSI is assumed. As expected, the performance degrades as \( \Delta f \sigma_d \) decreases. However, for \( \Delta f \sigma_d \) as small as 0.3, the performance is very close to the case of independent fading subcarriers (\( \Delta f \sigma_d = \infty \)). Observe that the BER gap between the LDD and SIC narrows as \( \Delta f \sigma_d \) decreases in Fig. 3 since fading, not MAI, becomes the dominant source of performance degradation as diversity reduces for small \( \Delta f \sigma_d \). Similarly, unreliable predicted CSI narrows the performance gap between these two detectors in the adaptive system [9]. Since the allocation algorithms for SIC and LDD have similar complexity [9], choosing the LDD at the receiver provides the best complexity/performance/robustness trade-off in this adaptive system.

In Fig. 4 and 5, we investigate a practical system design with the available bandwidth set to 1.25MHz, the bandwidth of single carrier in IS-95 and CDMA2000 standard. The rms delay spread of the channel is assumed \( \sigma_d = 1 \) us, which is typical for the urban environment [19, pp.200]. Since the coherence bandwidth is approximately \( 1/5\sigma_d = 200 \) KHz [19, pp.202], we divide the available bandwidth into 12 subcarriers with the subcarrier bandwidth of 102.4 KHz to assure flat fading for each subcarrier. Assuming \( N=8 \) substreams and \( PG = 64 \), the system supports a 12.8 Kbps data service to each user. Moreover, for comparison, we have simulated a non-adaptive MC DS-CDMA system with the same system bandwidth, number of subcarriers, and data rate as for the adaptive system. To maintain the data rate, the time-domain processing gain of eight is required. While in original MC DS-CDMA design in [6], the same spreading code was used for all subcarriers of one user, we employ different random spreading codes for different subcarriers as proposed in [20] to reduce the variance of the cross-correlation between the waveforms of different users.

Fig. 4 shows the BER of the adaptive system for varying numbers of users. For all detectors, the BER level is maintained provided the number of users \( K \) is sufficiently small. This is due to the fact that the fading induced near-far problem can be easily suppressed for a small number of users. While the BER of the MF receiver degrades for \( K>M \), the MUDs remain reliable performance unless the system load is very large, i. e., when \( K>M \), and the LDD has the lowest BER.

In Fig. 5, the BERs of the adaptive and non-adaptive systems with the LDD are compared. For the non-adaptive system, this MUD is derived from the RAKE LDD designed for multipath channel in [21]. Even with poorly predicted CSI (\( \eta = 0.925 \)), the AFH system outperforms the non-adaptive system. For target BER of \( 10^{-3} \), the adaptive system can support from 12 to 63 users, depending on the quality of the predicted CSI, while only two users are feasible for the non-adaptive system.

V. CONCLUSIONS

In this paper, the performance of an improved allocation algorithm for MC-CDMA system with AFH is investigated. Even in the presence of imperfect CSI at the base-station, the combined allocation algorithm and MUD are very efficient in mitigating MAI, resulting in much larger system capacity than that of the non-adaptive MC DS-CDMA system. This conclusion is confirmed for realistic fading channels with correlated subcarriers and practical system parameters.

REFERENCES

Fig. 2: Comparison of SUB of non-adaptive and adaptive systems with the BER of MC-CDMA with AFH and MUD for $K=16$ users, $N=8$ substreams/user, and varying CSI reliability $\eta$. In all systems, $PG=64$, $M=8$ subcarriers and $\Delta f_{\sigma_d}=\infty$

Fig. 3: BER performance of MC-CDMA with AFH and LDD/SIC for $K=16$, $M=8$, $N=8$, $PG=64$, and perfect CSI at the allocation algorithm.

Fig. 4: BER vs. number of users in the adaptive system with improved allocation algorithm for $SNR=10dB$, $M=12$, $N=8$, $PG=64$, $\eta=0.95$, and $\Delta f_{\sigma_d}=0.1024$.

Fig. 5: BER vs. number of users in the system with LDD for $SNR=10dB$, $M=12$, $N=8$, $PG=64$, and $\Delta f_{\sigma_d}=0.1024$.


