Reliable Adaptive Modulation Using Long-Range Prediction at a Different Carrier Frequency

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Abstract—Adaptive transmission techniques, such as adaptive modulation and coding, adaptive power control, adaptive transmitter antenna diversity, etc., generally require precise channel estimation and channel state information feedback. For fast vehicle speeds, reliable adaptive transmission also requires prediction of future channel state information (CSI) since the channel conditions are rapidly time-variant. In this paper, we propose to use past channel observations of one carrier to predict future CSI and perform adaptive modulation without feedback for another correlated carrier. Statistical model of the prediction error that depends on the frequency and time correlation is developed and is used in the design of reliable adaptive modulation methods. Significant gains relative to non-adaptive techniques are demonstrated for sufficiently correlated channels and realistic prediction range. Both Jakes fading model and a novel realistic physical model of frequency selective fading are used to validate performance of the proposed method.

1. INTRODUCTION

High-speed wireless communications require robust channel estimation and adaptive transmission to satisfy the tremendous growth in demand for capacity. The idea of adaptive transmission [1-4] is to change the transmission parameters according to the instantaneous fading channel power without sacrificing bit-error rate (BER). These schemes can provide higher bit rates relative to conventional signaling by transmitting at high rate under favorable channel conditions, and reducing throughput as the channel degrades. These adaptive modulation methods depend on accurate channel state information (CSI) that can be acquired from different sources. If the communication between the two stations is bi-direction and the channel can be considered reciprocal, then each station can estimate the channel quality on the basis of the received symbols and adapt the parameters to this estimation. This is called open-loop adaptation [5]. If the channel is not reciprocal, the receiver has to estimate the channel quality from feedback resulting in closed-loop adaptation. The feedback delay, overhead, and processing delay will degrade the system performance of the adaptive modulation, especially in rapidly time variant fading. Even in open-loop channels, current CSI is not sufficient since future channel conditions need to be known to adapt transmission parameters. To realize the potential of adaptive transmission methods, the channel variations have to be reliably predicted at least several milliseconds ahead.

Recently, a novel adaptive long-range prediction method was proposed in [6-11]. The algorithm employs an autoregressive (AR) model to characterize the fading channel and computes the minimum mean-square-error (MMSE) estimate of a future fading coefficient based on a number of past observations. The advantage of this algorithm relative to conventional methods is due to its low sampling rate (on the order of twice the maximum Doppler shift and much lower than the data rate), which results in longer memory span and further prediction into the future for a fixed filter length.

In this paper, we extend the long-range prediction algorithm into frequency domain. In particular, we concentrate on the scenario where we observe a received uplink signal at the carrier frequency $f^1$ and attempt to predict the downlink signal at the carrier frequency $f^2$ without feedback from the mobile. Alternatively, a signal at frequency $f^1$ can be fed back and a signal at adjacent frequency $f^2$ is predicted without feedback. To accomplish this prediction, the predicted samples must be sufficiently correlated with the observations in both time and frequency. This technique can be applied in correlated uplink and downlink channels, in orthogonal frequency division multiplexing (OFDM) systems (where narrow correlated sub-channels are employed) or other wideband systems to reduce feedback and overhead requirements.

The remainder of this paper is organized as follows. In the next section, we present the system model and describe the long-range prediction technique, the statistical model of the prediction error and channel statistics. In section 3, the adaptive modulation scheme is discussed and the performance of adaptive modulation aided by long-range prediction is demonstrated.

2. System Model and Channel Statistics

The discrete-time system model is illustrated in Fig. 1. The frequency of observed CSI is $f^1$ and the frequency of transmitted signal is $f^2$. Let $c(f^1,n)$, $n=1,2$, be samples of the fading signal $c(f^1,t)$ at the sampling interval $T_s$. Assume stationary and ergodic time-varying complex channel gain sequence $c(f^1,n)$ with distribution $p_c(x)$. We assume that $E[c(f^1,n)]=1$. The linear MMSE prediction of the future channel sample $c(f^1,n)$ at frequency $f^2$ based on $p$ previously observed samples $c(f^1,n-j)$ at frequency $f^1$ is given by:

$$\hat{c}(f^2,n) = \sum_{j=1}^{p} \mathbf{d}_j c(f^1,n-j)$$

The optimal coefficients $\mathbf{d}_j$ are determined as:

$$\mathbf{d} = \mathbf{R}^{-1} \mathbf{r}$$

where $\mathbf{d}=(d_1,\ldots,d_p)^T$. $\mathbf{R}$ is the autocorrelation matrix $(p_x p)$ with coefficients $R_{ij} = E[c(f^1,n) c^*(f^1,n-j)]$ and $\mathbf{r}$ is the autocorrelation vector $(p_x 1)$ with coefficients $r_j = E[c(f^1,n) c^*(f^1,n-j)]$. The resulting MMSE is given by:
In practice, the samples $c(f_i, n)$ are observed in the presence of additive white Gaussian noise (AWGN) $n(i)$ with power spectrum density (PSD) $N_0$. Equations (1-3) can be easily modified to include noisy observations [7]. Throughout the paper, we employ $p = 100$, the sampling rate of 500Hz and the observation interval of 100 samples assuming the maximum Doppler shift of 100Hz. The performance of the system is obviously dependent on the time and frequency correlation function that will be discussed next.

The statistics of fading signals received at correlated carriers are discussed in [12]. The fading coefficients at two frequencies can be expressed as:

$$c(f_1, t) = \sum_{n=1}^{N} A_n e^{j(2\pi f_1 t + f_1 n)}$$

$$c(f_2, t) = \sum_{n=1}^{N} A_n e^{j(2\pi f_2 t + f_2 n)},$$

where for the $n$\textsuperscript{th} path, $A_n$ is the (real) amplitude and $f_n$ is the Doppler shift. The phase difference of the $n$\textsuperscript{th} path $f_{2n} - f_{1n} = 2\pi Df T_n$ where $Df = f_2 - f_1$ is the frequency separation, and $T_n$ is the delay spread. For large $N$, $c(f^*, t)$ is distributed approximately as a zero mean complex Gaussian random variable. Hence the amplitudes $A_1(t) = |c(f_1, t)|$ and $A_2(t) = |c(f_2, t)|$ are both Rayleigh distributed. Assume angular distribution of the incident power is uniform between $[0, 2\pi]$, horizontal directivity pattern of the receiving antenna is 1, and the delay spread $T_n$ is exponentially distributed [12] with the probability density function (pdf):

$$p(T) = \frac{1}{\sigma e^{-T/\sigma}},$$

where $\sigma$ is a measure (rms delay spread [14]) of the time delay spread. The cross-correlation of the two fading signals with the time difference $\tau = |t_1 - t_2|$ and the frequency separation $\Delta f = f^2 - f^1$ can be derived as:

$$R(\tau, T) = E[c(f_1, t) c^*(f_2, t+\tau)] = R(\tau) R(T)$$

$$R(T) = J_0(2\pi T f_d)$$

where $R(T) = 1 + (2\pi T f_d)^2 - j2\pi T f_d / (1 + (2\pi T f_d)^2).$
computation of the accuracy factor for the prediction method proposed in this paper.

Once the channel statistics, such as the time and frequency domain correlation, are known, the optimum MMSE channel prediction can be employed as in (1-2). However, in mobile wireless links the channel autocorrelation function $R_t(\tau)$ (6) depends on the particular environment and changes with time. Therefore we develop a system that can adapt its parameters to the time-variant autocorrelation function. The basic idea is to predict future channel coefficient $c(f_1,n)$ first and then to use the frequency correlation function to select the transmitter parameter at $f_2$. The future predicted CSI at $f_1$ are given by:

$$\hat{c}(f_1,n) = \sum_{j=1}^{p} g_j c(f_1,n-j) .$$

(9)

The coefficients $g_j$ are determined using the Least Mean Square (LMS) adaptive tracking method:

$$g(n) = g(n-1) + \mu e c(f_1,n) ,$$

(10)

where $\mu$ is the step size and $e = c(f_1,n) - \hat{c}(f_1,n)$. This adaptive tracking can be performed since the observations at frequency $f_1$ are available at the transmitter [7,8,10]. Recursive least square (RLS) can also be used to improve accuracy and reduce the observation interval [15]. Once $c(f_1,n)$ is found, the adaptive modulation parameters for transmitting at $f_2$ at time $n$ are selected as explained in section 3. (Note that $c(f_2,n)$ is not predicted directly). This procedure depends on the pdf of the accuracy factor $\beta = \frac{\hat{\alpha}(f_1,t)}{\alpha(f_1,t)}$, where the prediction at frequency $f_1$ at time $t$ serves as an estimate of $\alpha(f_1,t)$. In Fig. 3, we compare the ideal $\hat{\beta} = \frac{\hat{\alpha}(f_1,t)}{\alpha(f_1,t)}$ obtained assuming perfect prediction at frequency $f_1$, and $\beta$ for $\alpha(f_1,t)$ predicted 2 ms (one step) ahead for $f_{as} = 100$Hz and two different values of normalized frequency separation. The pdf of $\beta$ is given by (8) with $\lambda = 1$ and $\rho = 1/(1+2\pi f_0^2)$. The pdf of $\hat{\beta}$ is obtained by numerical estimation of $\rho$ and $\lambda$ in (8).

### 3. Adaptive Transmission Using Long Range Channel Prediction

In this paper, we employ variable rate and variable power square M-QAM signal constellations due to their inherent spectral efficiency and ease of implementation [13]. Given fixed transmitter power per symbol $E_s$ (or average SNR level $\gamma = E_s/N_0$) and a target bit error rate $BER_{as}$, we adjust the modulation level $M$ according to the instantaneous channel gain $\alpha_i$. A decision rule for the modulation level selection based on prediction accuracy factor was proposed in [9]. Assume $\hat{\alpha} = \alpha(f_1,t)$ and $p_{b}(x)$ is given by (8). The thresholds $\alpha_i$, $i = 1...4$ are chosen as follows. When the predicted channel gain $\alpha_i \geq \hat{\alpha} \geq \alpha_M$, $M(i)$-QAM is employed, where $M(1) = 2, M(i) = 2^{2(i-1)}$, $i = 2...4$, $(\alpha_i = \infty)$ based on the equation:

$$BER_{M(i)*}(\hat{\gamma},\hat{\alpha}) = \int_0^\infty BER_{M(i)}(\gamma,\hat{\alpha})p_{b}(x) dx \leq BER_{as} ,$$

(11)

where $p_{b}(x)$ is described by (8). We can calculate $BER_{as}$ from the BER bound of MQAM for an AWGN channel [1]:

$$BER_{as}(\gamma) \leq 0.2\exp(-1.5\gamma(M(i)-1)) \quad M(i) \geq 4$$

(12)

$$BER_{as}(\gamma) = Q(\sqrt{2\gamma}) ,$$

where $\gamma$ is the signal-to-noise ratio per symbol. Once the threshold and the constellation are fixed, the BER of this fixed power discrete rate adaptive modulation is lower than the BER of the constellation $M(i)$ since the thresholds $\alpha_i$ are chosen using the upper bound in (11). However, we can use a power control policy that satisfies the target BER. From (11), a fixed BER for the constellation $M(i)$ can be maintained by multiplying the power $E_s$ by $(\alpha_i/\hat{\alpha})^2$. Then $BER_{as}(\gamma,\hat{\alpha})$ becomes:

$$BER_{as}(\gamma,\hat{\alpha}) = \int_0^\infty BER_{M(i)}(\gamma,\hat{\alpha})p_{b}(x) dx = BER_{as}$$

(13)

Another variable rate and variable power adaptive transmission was proposed in [1]. The simpler method described in this paper results in less than 0.5dB power loss compared to [1]. The average bit per symbol $R_{as}$ for both fixed and variable power methods is:
\[
\hat{R}_{as} = \sum_{i=1}^{\alpha_i} \log_2 M_i \int_{\alpha_i}^{\alpha_{i+1}} p_{\alpha}(x) dx
\]  

(14)

\[
P_{as} = \sum_{i=1}^{\alpha_i} \int_{\alpha_i}^{\alpha_{i+1}} E_\alpha(x) p_{\alpha}(x) dx .
\]  

(15)

where the pdf of predicted amplitude \( p_{\alpha}(x) \) is:

\[
p_{\alpha}(x) = \frac{2x}{\Omega} \exp(-\frac{x^2}{\Omega}).
\]  

(16)

Fig. 4. Bit per symbol vs. \( \rho \) for different SNR for power control M-QAM. Target BER=10^{-3}, \( \lambda = 1 \).

This rate also gives the spectral efficiency assuming the ideal Nyquist data pulse. For the power control method above, the average transmission power \( P_{as} \) is

\[
P_{as} = \sum_{i=1}^{\alpha_i} \int_{\alpha_i}^{\alpha_{i+1}} E_\alpha(x) p_{\alpha}(x) dx .
\]  

(15)

where the pdf of predicted amplitude \( p_{\alpha}(x) \) is:

\[
p_{\alpha}(x) = \frac{2x}{\Omega} \exp(-\frac{x^2}{\Omega}).
\]  

(16)

For the power control adaptive modulation method described above, we plot the BPS (14) vs. the correlation \( \rho \) in (8) for different SNR computed from (15) with \( \lambda = 1 \) and BER = 10^{-3} in Fig. 4. The correlation \( \rho = 1 \) corresponds to perfect prediction, while \( \rho = 0 \) represents the worst case when the BPS of the adaptive modulation converges to that of the non-adaptive M-QAM for given SNR and BER.<br>

The power control method above involves continuously varying transmitter power. It can be simplified by selecting a constant power for each constellation \( M_i \) level to maintain the target BER. The power \( E_\alpha(i) \) for each constellation \( M(i) \) is chosen using:

\[
E_\alpha(i) = \int_{\alpha_i}^{\alpha_{i+1}} \frac{E_{\alpha}(\hat{\alpha}, \alpha)}{N_0} \rho(\hat{\alpha}|\alpha, \alpha_i < \hat{\alpha} < \alpha_{i+1}) d\hat{\alpha} \leq \text{BER}_{tg}.
\]  

(17)

where BER\(_{M(i)}\) is computed in (11) and \( \rho(\hat{\alpha}|\alpha, \alpha_i < \hat{\alpha} < \alpha_{i+1}) \) is the conditional pdf of \( \hat{\alpha} \) determined using thresholds \( \alpha_i \) also chosen using (11). Thus each modulation level is only associated with one transmit power. This is called discrete rate discrete power [1].

We compare several adaptive transmission techniques in Fig. 5. Perfect CSI is assumed. Continuous rate and power adaptation [1] is included in the comparison. Note that the method places no restrictions on the constellation size, which makes it impractical. We also found that the fixed-rate truncated channel inversion [1] based on M-QAM has similar performance to the discrete power discrete rate method, while it is also non-feasible in practice. We also plot the Shannon capacity of the fading channel [1] for comparison. We observe that our continuous power control policy achieves about 3dB gain relative to the fixed power discrete rate adaptive modulation, and the discrete rate discrete power method has power loss of less than 2dB relative to the continuous power discrete rate transmission scheme.

Finally, we illustrate the performance of continuous power discrete rate adaptive M-QAM aided by channel prediction in Fig. 6. The target BER = 10^{-3}. The symbol rate is 25ksymbol/s, and the modulation-switching rate is set to the symbol rate. Interpolation is utilized to predict the channel coefficients at the symbol rate. In Fig. 6 we plot bits per symbol vs. normalized frequency separation \( \Delta f \sigma \) for the ideal (non-adaptive) MMSE filter (1-2) and the robust method using LMS (9-10) algorithm with step size 0.005 and estimated \( \rho = 0.97 \) and \( \lambda = 0.995 \). We also plot the BPS of the robust method (9-10) assuming that prediction is perfect at frequency \( f^1 \). We observe that the performance in this case is close to that of the optimal MMSE method. Hence the robust method is nearly optimal and has the ability to adapt transmission parameters to the time-variant channel conditions.

Fig. 6 also shows that adaptive modulation is primarily beneficial when normalized frequency separation \( \Delta f \sigma \) does not significantly exceed 0.1. Beyond this \( \Delta f \sigma \), the BPS approaches that of non-adaptive transmission case. For example, for \( \Delta f \sigma = 0.1 \) about 17 dB is required to obtain 1 BPS for adaptive M-QAM as opposed to 24 dB for non-adaptive transmission. As \( \Delta f \sigma \) approaches 0.4, the bit rate of

Fig. 5. Spectral efficiency vs. SNR for different transmission techniques. Target BER = 10^{-3}.
adaptive modulation is very close to that of non-adaptive transmission. Hence the frequency separation and the multipath delay (or the coherence bandwidth) are the factors that determine the performance of the proposed adaptive modulation method.

The typical values of $\sigma$ are on the order of microseconds in outdoor mobile radio channel [14]. Suppose $\Delta f\sigma = 0.1$ and $\sigma = 1\mu$ sec. Then the frequency separation $\Delta f = 100$KHz. This means that two channels can be separated by 100KHz and still benefit from the proposed adaptive transmission method. If $\sigma = 10\mu$ sec, smaller $\Delta f \leq 10$KHz will results in good spectral efficiency when adaptive transmission is used.

The results presented above assumed stationary frequency selective fading environment created using the Jakes model [12]. We also validate performance of the proposed method using a novel physical multipath fading model that includes the variation of parameters associated with different reflectors. This model is a generalization of the physical model of flat fading presented in [10, 15]. Performance and robustness of the proposed adaptive modulation method for various scattering environments are addressed in the presentation.

**4. Conclusion**

We presented a novel adaptive modulation method that uses predicted CSI of a different carrier. The statistical model of the prediction accuracy factor was created and system performance was evaluated for various frequency separation values. We showed that increased frequency separation and multipath delay limit the performance of this system. A novel realistic physical frequency selective channel model was introduced to validate performance for diverse scattering environments. The results can be applied in OFDM systems or in correlated uplink and downlink channels to reduce feedback and overhead requirements.

**Fig.6. BPS vs. normalized frequency separation for different prediction techniques. $f_{\text{im}} = 100$Hz.**

**REFERENCES**