

# Reliable Adaptive Modulation and Interference Mitigation for Mobile Radio Slow Frequency Hopping Channels<sup>1</sup>

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**Abstract:** The Long Range Fading Prediction algorithm for Slow Frequency Hopping (SFH) systems is proposed and demonstrated to enable combined Adaptive Modulation and adaptive Frequency Diversity to mitigate the effects of fading and partial-band interference. Significant performance gains are demonstrated relative to non-adaptive methods in realistic mobile radio SFH channels where the total bandwidth does not exceed approximately 15 times the coherence bandwidth.

**Key words:** Slow Frequency Hopping, Channel State Information, Long Range Prediction, Adaptive Transmission, Partial-band Interference, Diversity.

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## 1. Introduction

Fading prediction methods for mobile radio channels were shown to enable adaptive modulation AM [1-3] in narrowband, Orthogonal Frequency Division Modulation (OFDM), adjacent frequency and Multiple Input Multiple Output (MIMO) fading channels in, e.g., [4-8,10,25-27]. In this letter and [9,11,24], we explore AM aided by the Long range prediction (LRP) for Slow Frequency Hopping (SFH) spread spectrum mobile radio systems that employ coherent detection [13,15]. We propose to predict the channel coefficients in the next hopping frequency of SFH systems based on a number of past fading observations from previous hopping frequencies. Fading prediction is challenging in this case since past observations are at different frequency slots constrained by the hopping pattern. Moreover, we investigate joint adaptive transmission that combines frequency diversity and AM to mitigate the effects of partial-band interference and fading in SFH systems with coherent detection.

## 2. System Model and Long Range Prediction for SFH Channels

Consider the SFH system that employs coherent detection [15,16] with the total number of frequencies  $q$ , the hopping rate  $f_h$ , and the frequency separation between adjacent carrier frequencies  $\Delta f$ . In this letter, we employ a randomly chosen periodic hopping pattern with length  $N=q$ , although the proposed methods also apply to non-periodic hopping patterns. Let  $c(f,t)$  be the equivalent lowpass complex sample of the flat fading channel at time  $t$  and frequency  $f$ , where  $f$  is the carrier frequency (slot) occupied at time  $t$  [13]. The channel coefficient  $c(f,t)$  is closely approximated by a zero mean complex Gaussian random process with Rayleigh distributed amplitude and uniformly distributed phase [12], and we assume  $E|c(f,t)|^2=1$ . The spaced-time spaced-frequency correlation function with the time difference  $\tau$  and the frequency separation  $\Delta f$  is defined as [7,12,13,22]:

$$R(\Delta f, \tau) = E[c(f,t)c^*(f+\Delta f, t+\tau)] = R_t(\tau)R_f(\Delta f), \quad (1)$$

where the factors in the last expression are the time and frequency correlation functions.

Figure 1 illustrates the adaptive transmission aided by the LRP for this FH system. Past reliable observations from all frequencies are fed back from the receiver to the transmitter. The transmitter employs the LRP to predict future Channel State Information (CSI), and adapts the transmission parameters to the channel variation. We employ the Minimum Mean Square Error (MMSE) linear prediction (LP) algorithm. Assume the channel coefficients  $c(f,t)$  are sampled at the rate  $f_s=1/T_s$ , and

for an integer  $n$ , define  $c(f(n),n)=c(f(nT_s),nT_s)$ . The prediction  $\hat{c}(f(n+\tau),n+\tau)$  ( $\tau$  is a positive integer) of the future channel coefficient  $c(f(n+\tau),n+\tau)$  based on  $p$  past observations  $c(f(n),n),\dots, c(f(n-p+1),n-p+1)$  is formed as (see Figure 1b)

$$\hat{c}(f(n+\tau),n+\tau)=\sum_{j=0}^{p-1}d_j(n)c(f(n-j),n-j) \quad (2)$$

where  $d_j(n)$  are the filter coefficients at time  $n$ , and  $\tau T_s$  is the prediction range. Note that the sampling rate in (2) is much slower than the symbol rate, but faster than the hopping rate  $f_h$ .

The objective is to find the LP coefficients that minimize the MSE, defined as  $E[|e(n)|^2]=E[|c(f(n+\tau),n+\tau)-\hat{c}(f(n+\tau),n+\tau)|^2]$ . Because the hopping pattern is a random frequency sequence, a single prediction filter does not exist, and the LP coefficients need to be re-computed at the sampling rate. The optimal LP filter used at sampling time  $n$  is given by [13]  $\mathbf{d}(n)=\mathbf{R}(n)^{-1}\mathbf{r}(n)$ , where  $\mathbf{d}(n)=[d_0(n)\dots d_{p-1}(n)]^T$ ,  $\mathbf{R}(n)$  is the  $p\times p$  autocorrelation matrix of the observations at time  $n$  with  $R_{ij}(n)=E\{c(f(n-i),n-i)c^*(f(n-j),n-j)\}$ , and  $\mathbf{r}(n)$  is the cross-correlation vector of the observations and the prediction at time  $n$  given by  $r_j(n)=E\{c(f(n+\tau),n+\tau)c^*(f(n-j),n-j)\}$ ,  $i,j=0,1,\dots,p-1$ , for given  $\tau$ . Both  $\mathbf{R}(n)$  and  $\mathbf{r}(n)$  are determined by the correlation function (1). The effect of additive noise on the observations can be incorporated in (2) [4]. In this letter, we assume that the noise in the observation samples is negligible. In practice, noise reduction techniques can be employed to improve the accuracy of the prediction [4,26,27]. For realistic SFH systems, the prediction MSE loss is dominated by observations constrained to the hopping pattern [9,11,24], and the degradation due to additive noise is relatively small in this MSE region [27].

For realistic mobile radio channels, the correlation functions  $R_t(\tau)$  and  $R_f(\Delta f)$  in (1) must be estimated and updated when new observations become available. We employ pilot symbols for estimation [11]. The rate of update of the correlation function estimates and the computational load of the estimation is low for realistic mobile radio channels [7,11]. On the other hand, the optimal MMSE channel prediction method (2) is complex (on the order of  $p^3$  multiplications [28]), because it requires inversion of a large matrix at the sampling rate. To reduce the complexity to the order of  $p^2$ , while maintaining the same performance, we employ a recursive procedure for updating this inverse as described in [9,11]. While it is possible to reduce complexity further by employing, e.g.,

the simplified LRP method in [9], we have observed that suboptimal prediction methods greatly degrade performance of adaptive FH systems.

### 3. Adaptive Modulation Aided by LRP

LRP is employed to enable AM for each upcoming dwell interval. We employ adaptive discrete power discrete rate MQAM with  $M=2, 2^{2^{(i-1)}}$ ,  $i=2,3,4$  [1]. As in [3,24,26], reliable performance is maintained by incorporating the accuracy of the predicted CSI into the AM design. The symbol rate is 20Ksps (symbols per second), and the target Bit Error Rate ( $BER_{tg}$ ) is  $10^{-3}$ . The SNR is defined as the ratio of the average transmitted symbol power  $E_s$  to the complex white noise power spectral density  $N_0$  [1]. While performance with and without prediction is usually compared in other adaptive systems (e.g narrowband or OFDM [4,8]), this comparison is meaningless in adaptive SFH systems. The delay associated with the feedback and other system constraints is comparable with the dwell interval duration, so the channel estimates obtained during current dwell interval cannot be employed. A single outdated estimate at the previous hopping frequency is not helpful for enabling AM. Thus, the only practical alternative to using fading prediction for SFH is to resort to non-adaptive modulation. Therefore, we compare the Bit Per Symbol (BPS) of AM enabled by the LRP with that of non-adaptive MQAM.

In the numerical results, we employ the standard Jakes fading model, a typical slow hopping rate  $f_h=500$  hops/second, the number of frequencies  $q=32$ , and the feedback delay of at least 1ms. To enable AM, several samples are predicted for each dwell interval, and the average prediction range is 2ms. The maximum Doppler shift is 50Hz (equivalently, the prediction of 0.1 wavelengths ahead is illustrated). In LRP, we employ the near-optimal sampling rate 2 kHz and  $p=50$  in (2) [11].

In Figure 2, the spectral efficiency (BPS) of AM vs. average SNR is shown. We observe that significant gain can be achieved relative to non-adaptive modulation (Binary and Quaternary Phase Shift Keying (BPSK, QPSK)). The gain depends on the normalized frequency separation  $\Delta f\sigma$  given by the product of the frequency separation between two adjacent hopping frequencies and the rms delay spread  $\sigma$  of the fading channel [12-14]. In [9,11,24], we have demonstrated that the prediction MMSE increases as  $\Delta f\sigma$  grows since the observations and the prediction become less correlated. This dependency affects the bit rate of AM as shown in Fig. 2. For small  $\Delta f\sigma=0.01$ , the BPS with prediction approaches that with perfect CSI [8]. While the BPS diminishes as  $\Delta f\sigma$  increases, even

for large  $\Delta f\sigma=0.1$ , the gain relative to the non-adaptive modulation is about 3dB, or 1 BPS. A physical model proposed in [4-6] was used in [11,24] to investigate the performance of AM for SFH systems in realistic fading channels. While the BPS is lower for the physical model than for the Jakes model due to the time-variant correlation function (1), it has been demonstrated that significant improvement is still achieved relative to non-adaptive modulation.

We observe that the FH system benefits from adaptive transmission primarily when  $\Delta f\sigma$  does not significantly exceed 0.1. Suppose  $\sigma$  is 1 $\mu$ s, representative of suburban areas [12]. Then a SFH system would benefit from adaptive transmission when the frequency separation is as large as 100 KHz ( $\Delta f\sigma\approx 0.1$ ). In realistic SFH systems [13-16], the symbol rate is on the order of tens Ksps, and, thus, adaptive transmission aided by the proposed channel prediction method is feasible for these systems. More generally, we have found that AM is feasible in SFH systems when  $f_{dm}\leq 100$ Hz, and the total normalized bandwidth (TNB)  $q\Delta f\sigma$  is on the order of 3 or lower, or, equivalently, the total bandwidth  $q\Delta f$  does not exceed approximately 15 times the coherence bandwidth  $B_c\approx 1/5\sigma$  [14]. As the TNB grows, the spectral efficiency of AM saturates and approaches that of non-adaptive modulation. On the other hand, frequency diversity is usually exploited in FH communications [13], and its benefit increases as the TNB grows. Thus, adaptive transmission and diversity combining compliment each other over the practical range of frequency correlations in SFH systems.

Our results demonstrate that fading prediction is less accurate for SFH systems than for narrowband transmission [4,26], OFDM [8], direct sequence CDMA [4] and even when the observations are at an adjacent frequency [7]. This loss is due to the fact that the observations are constrained by the hopping pattern in LRP for SFH systems, and, thus, are widely distributed in frequency. This constraint degrades prediction accuracy, and hampers utilization of fast and efficient adaptive tracking techniques. However, we note that fading prediction is critical in SFH applications, since adaptive transmission would not be possible without prediction in FH systems.

#### **4. Adaptive SFH Systems with Partial-band Interference**

We focus on the Partial-Band Interference (PBI) that is not due to a hostile jammer [13,17,18]. It is usually slowly varying and modeled as narrow-band additive Gaussian noise with the average power spectral density  $N_I$  with that occupies a small fraction  $\delta$  of the total bandwidth of the FH

system. We use adaptive frequency diversity to jointly mitigate the effect of PBI and fading. In the proposed *diversity FH* method, the same information is transmitted on several carrier frequencies chosen according to a hopping pattern, and the outputs of different diversity branches are combined at the receiver. For simplicity, in this letter we employ only two frequencies (diversity branches)  $f^1$  and  $f^2$  with large separation  $N\Delta f/2$  and negligible correlation assumed assured by the appropriate hopping pattern design [11,20].

We assume that the receiver knows perfectly where the PBI is present [11,13,19]. At the transmitter, the uncertainty of PBI presence for two upcoming frequencies  $f^1$  and  $f^2$  is modeled as follows. Define the indicator function for the presence of PBI at the upcoming frequency  $f^k$ ,  $k=1,2$ , as  $I_k=1$  if the interference is present at  $f^k$ , and 0 otherwise. Given the reliability factor  $\eta \in [0,1]$ , the probability of the interference at the transmitter is modeled as

$$p_k = \eta I_k + (1-\eta)(1-I_k). \quad (3)$$

The following near-optimal diversity combining technique [11,13,21] is employed at the receiver. When there is no interference at both hopping frequencies  $f^1$  and  $f^2$ , Maximal Ratio Combining (MRC) is used. When only one frequency has interference, the PBI-free branch is selected using Selective Combining (SC). If both frequencies are interfered, random guess is used to detect data. More complex *sentient FH* technique (similar to the selective transmitter diversity [10]) can further improve performance. In this method, channel coefficients of  $L$  widely spaced frequencies (chosen according to the hopping pattern) are predicted, and a subset of  $r$  frequencies with the largest channel gains are selected in the transmission. Due to lower average transmitted power [11], sentient FH also results in lower Multiple Access Interference (MAI) than FH given the same number of transmitted frequencies  $r$ .

The degrading effect of PBI is compensated for by the benefit of frequency diversity on the performance of LRP for these diversity systems, and the prediction accuracy is better than for interference-free systems without diversity [11]. In AM for diversity FH, let  $\alpha_k$  and  $\hat{\alpha}_k$ ,  $k=1,2$ , be the actual and predicted fading channel amplitudes at the upcoming frequencies  $f^1$  and  $f^2$ , respectively. The average BER of fixed power AM when  $M(i)$ -QAM is employed by the transmitter

$$\begin{aligned} \text{BER}_{M(i)}^*(E_s, N_0, \hat{\alpha}_1, \hat{\alpha}_2, p_1, p_2) &= (1-p_1)(1-p_2) \int_0^\infty \int_0^\infty \text{BER}_{M(i)}(\gamma = \frac{E_s(\alpha_1^2 + \alpha_2^2)}{N_0}) p(\alpha_1 | \hat{\alpha}_1) p(\alpha_2 | \hat{\alpha}_2) d\alpha_1 d\alpha_2 \\ &+ p_2(1-p_1) \int_0^\infty \text{BER}_{M(i)}(\gamma = \frac{E_s \alpha_1^2}{N_0}) p(\alpha_1 | \hat{\alpha}_1) d\alpha_1 + p_1(1-p_2) \int_0^\infty \text{BER}_{M(i)}(\gamma = \frac{E_s \alpha_2^2}{N_0}) p(\alpha_2 | \hat{\alpha}_2) d\alpha_2 + 0.5p_1p_2, \end{aligned} \quad (4)$$

where the  $\text{BER}_{M(i)}$  is an upper bound on the BER of M(i)-QAM [1],  $p(\alpha_k | \hat{\alpha}_k)$  is the conditional pdf of  $\alpha_k$  given  $\hat{\alpha}_k$  [3], and  $p_k$  is given by (3),  $k=1,2$ . The modulation level is chosen as  $\tilde{M} = \max\{M(i) | \text{BER}_{M(i)}^*(E_s, N_0, \hat{\alpha}_1, \hat{\alpha}_2, p_1, p_2) \leq \text{BER}_{\text{tg}}\}$ . In AM combined with sentient FH, for  $r=2$ , (4) is employed, assuming  $f^1$  and  $f^2$  are the two frequencies (among L) with the largest prediction gains.

Simulations are used to demonstrate the performance of adaptive SFH for typical PBI values [23]. In Figure 3, the BPS of AM is illustrated under the assumption of perfect knowledge of PBI at the transmitter ( $\eta=1$ ). We observe that the BPS degrades as  $\delta$  increases, and sentient FH outperforms diversity FH. As L grows, the BPS of sentient FH saturates, and  $L=4$  has near-optimal performance [11]. We also show that the BPS of adaptive modulation for a non-diversity system (single Rayleigh fading channel) with PBI is poor [11]. (Note that when this method is extended to  $\eta < 1$ , the target BER cannot be satisfied, implying that diversity is required for channels with imperfect knowledge of PBI at the transmitter.) Figure 4 shows the BPS degradation of diversity FH as  $\eta$  decreases. For both diversity and sentient FH, when  $\eta \leq 0.95$ , the target BER cannot be satisfied with the adaptive transmission method proposed above, and additional diversity would be required to maintain reliable performance.

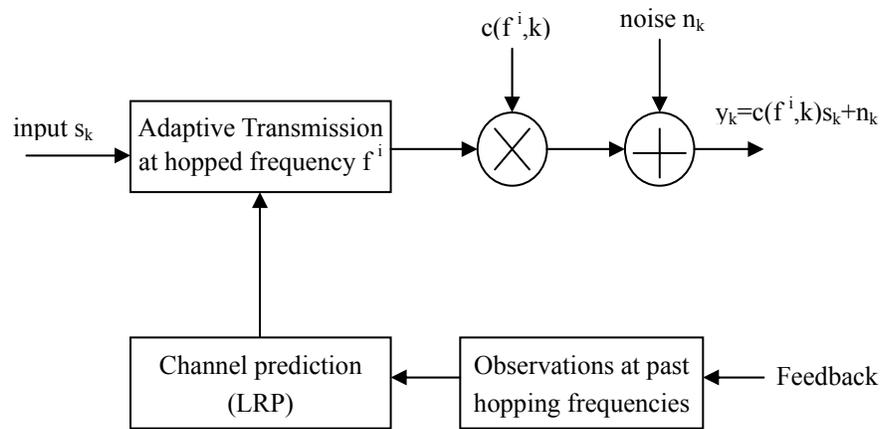
## 5. Conclusion

The optimal MMSE long range prediction algorithm for SFH communications with coherent detection was introduced. It was demonstrated that the proposed LRP method enables adaptive modulation for SFH. Moreover, joint adaptive frequency diversity and AM was investigated for channels with PBI. Numerical and simulation results demonstrate that significant performance gains can be achieved relative to non-adaptive modulation for realistic FH systems.

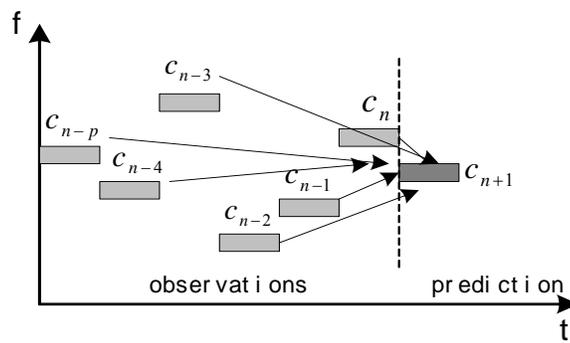
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a. System model



b. The LRP algorithm

Figure 1. Adaptive transmission for FH channels aided by long range prediction.

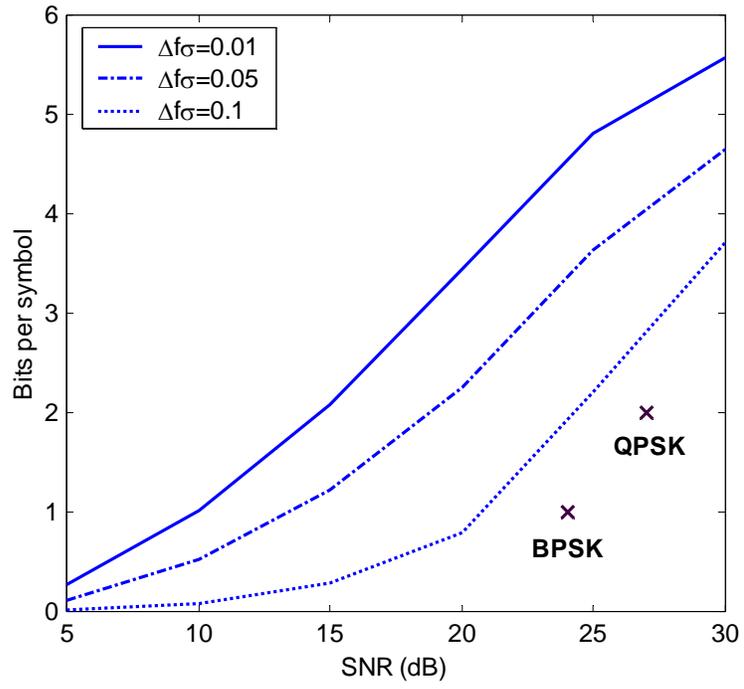


Figure 2. Spectral efficiency of adaptive modulation using Long Range Prediction,  $\tau T_s=2\text{ms}$ ,  
 $f_{dm}=50\text{Hz}$ ,  $q=32$ .

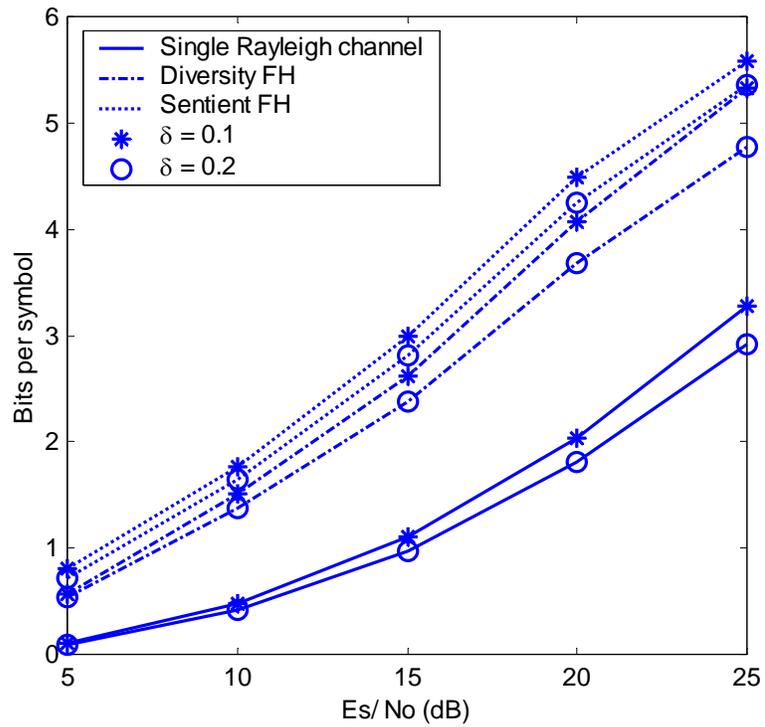


Figure 3. Performance of adaptive SFH with partial-band interference, Jakes model, prediction interval  $\tau T_s=2\text{ms}$ ,  $f_{dm}=50\text{Hz}$ ,  $\Delta f\sigma=0.05$ ,  $\delta=0.1$ ,  $\eta=1.0$ ,  $E_s/N_I=0\text{dB}$ ,  $\text{BER}_{\text{ig}}=10^{-3}$ ,  $L=4$ ,  $r=2$  for sentient FH.

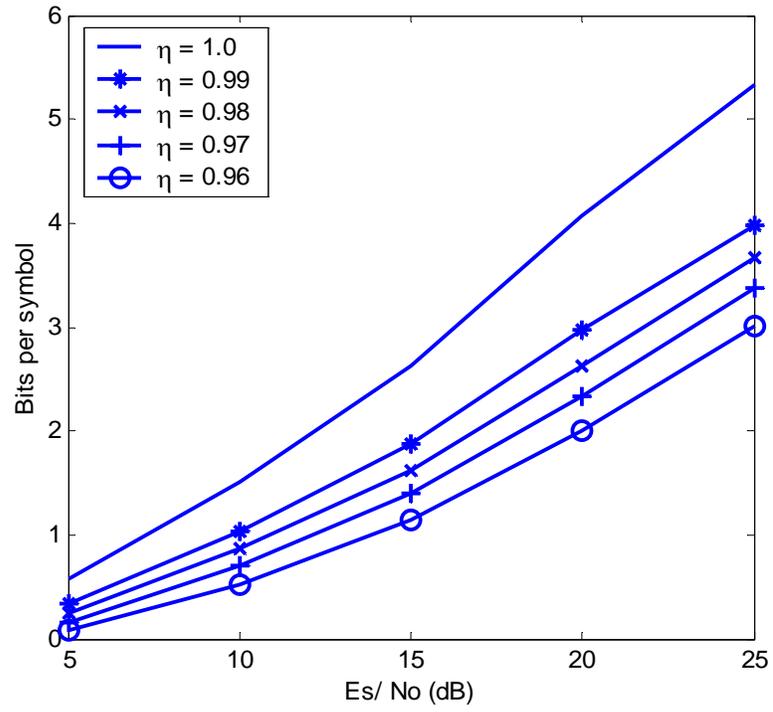


Figure 4. Performance of adaptive diversity FH with PBI, Jakes model,  $\delta=0.1$ ,  $\tau T_s=2\text{ms}$ ,  $f_{dm}=50\text{Hz}$ ,  $\Delta f\sigma=0.05$ ,  $E_s/N_1=0\text{dB}$ ,  $\text{BER}_{\text{targ}}=10^{-3}$ .