XQCD: Elliptic flow and heavy quarks

XSYM: Heavy quarks in AdS/CFT

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- Heavy quarks: Jorge Casalderrey-Solana, DT; hep-th/0701123
- Heavy quarks: Jorge Casalderrey-Solana, DT; hep-ph/0605199
- Heavy quarks: Jorge Casalderrey-Solana, D. T. Son; In progress
Observation:

There is a large momentum anisotropy:

\[ v_2 \equiv \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle} \approx 20\% \]

Interpretation

- The medium responds as a fluid to differences in \( X \) and \( Y \) pressure gradients

Hydro models “work”
Data on Elliptic Flow:

\[
\frac{1}{p_T} \frac{dN}{dp_T d\phi} = \frac{1}{p_T} \frac{dN}{dp_T} (1 + 2v_2(p_T) \cos(2\phi) + \ldots)
\]

\[\text{STAR Preliminary} \]

Elliptic flow is large \(X:Y \sim 2.0 : 1\)
Hydrodynamics:

- For hydrodynamics need:
  \[
  \frac{\ell_{\text{mfp}}}{R_{\text{Au}}} \ll 1
  \]

- How to define \( \ell_{\text{mfp}} \)?
  \[
  \ell_{\text{mfp}} \sim \frac{\eta}{e + p} \quad e + p = sT
  \]

Condition:

\[
\frac{\eta}{s} \quad \text{Medium Property} \sim 1/\alpha_s^2
\]

\[
\times \quad \frac{1}{R_{\text{Au}}T} \quad \text{Experimental Property} \sim 1/2
\]

Need \( \eta/s \) small.
When is Hydrodynamics Valid?

- Go out of equilibrium when expansion rate is too fast

\[ \tau_R \frac{\partial \mu u^\mu}{\partial t} \sim \frac{1}{2} \left( \frac{1}{V} \frac{dV}{dt} \right) \]

- The viscosity is related to the relaxation time

\[ \frac{\eta}{e} \sim v_{\text{th}}^2 \tau_R \quad p \sim e v_{\text{th}}^2 \]

- So the freezeout criterion is

\[ \frac{\eta}{p} \frac{\partial \mu u^\mu}{\partial t} \sim \frac{1}{2} \]
Hydrodynamic Simulations of Central Heavy Ion Collisions

Need $\eta/s$ small to describe a large fraction of collision
Solving Navier Stokes

- The Navier Stokes equations

\[ \partial_\mu T^{\mu\nu} = 0 \]
\[ T^{ij} = p\delta^{ij} + \pi^{ij} \]

\( T^{ij} \) = equilibrium \( \pi^{ij} \) = correction

- The “first order” stress tensor instantly assumes a definite form.

\[ \pi^{ij} = -\eta \left( \partial^i v^j + \partial^j v^i - \frac{2}{3} \delta^{ij} \partial \cdot v \right) \]
\[ O(\epsilon) = O(\epsilon) \]

- Can make “second order” models which relax to the correct form

\[ -\tau_R \partial_t \pi^{ij} = \pi^{ij} + \eta \left( \partial^i v^j + \partial^j v^i - \frac{2}{3} \delta^{ij} \partial \cdot v \right) \]
\[ O(\epsilon^2) = O(\epsilon) + O(\epsilon) \]

Can solve these models
Independent of second derivative terms

\[ \chi = 3 \quad \eta/s = 0.05 \]

Gradient expansion is working. Temperature is a good concept.

Worse at larger viscosities and larger \( p_T \)
Running Viscous Hydro

- Run the evolution and monitor the viscous terms

- When the viscous term is about half of the pressure:
  - \( T^{ij} \) is not asymptotic with \( \eta \langle \partial^i v^j \rangle \)

  Freezeout is signaled by the equations.

- Kinetic theory distribution Functions modified
  - With viscosity \( T^{\mu \nu} \rightarrow T^{\mu}_0 + \delta T^{\mu \nu} \) so \( f \rightarrow f_0 + \delta f \).

  Maximum \( p_T \) is also signaled by the equations.
Bjorken Solution with transverse expansion: \((\eta/s = 0.2)\)

Viscous corrections do NOT integrate to give an O(1) change to the flow.
Viscous corrections to the distribution function $f_o \rightarrow f_o + \delta f$

- Corrections to thermal distribution function $f_0 \rightarrow f_0 + \delta f$
  - Must be proportional to strains
  - Must be a scalar
  - General form in rest frame and ansatz

\[ \delta f = F(|\mathbf{p}|) \pi^i \pi^j \pi^{ij} \rightarrow \delta f \propto f_0 \mathbf{p}^i \mathbf{p}^j \pi^{ij} \]

- Can fix the constant

\[ p \delta^{ij} + \pi^{ij} = \int \frac{d^3 \mathbf{p}}{(2\pi)^3} \frac{p^i p^j}{E_\mathbf{p}} (f_0 + \delta f) \]

find

\[ \delta f = \frac{1}{2(e + p)T^2} f_0 \mathbf{p}^i \mathbf{p}^j \pi^{ij} \]
Viscous Hydro Results:

STAR Data
16-24% Central

\[ v_2(p_T) \]

\[ \chi=3 \]
Ideal
\[ \eta/s=0.05: f_0 + \delta f_\pi \]
\[ f_0 + \delta f_G \]
\[ \eta/s=0.2: f_0 + \delta f_\pi \]
\[ f_0 + \delta f_G \]
Effect of modifying the distribution function $\eta/s = 0.2$
Estimate the uncertainty between first order and "some" second order

\[ \pi_{ij} \approx \eta \langle \partial^i u^j \rangle + O(\epsilon^2) \]
Compare to $\eta/s = 0.05$. 

\[ \chi = 3 \]

Ideal

$\eta/s = 0.05$: $f_0$

$\eta/s = 0.2$: $f_0$

\[ \pi^{ij} \simeq \eta \left\langle \partial^i u^j \right\rangle + O(\epsilon^2) \]

\[ \pi^{ij} \]

$\eta \left\langle \partial^i u^j \right\rangle$
Viscous Hydro Results:

- To get anywhere close to the data need:
  \[ \frac{\eta}{s} \sim \frac{1}{4\pi} \]

- The hydrodynamic results are under relatively good control when
  \[ \frac{\eta}{s} \sim \frac{1}{4\pi} \]
View heavy quark energy loss as Brownian Motion
Experimental Motivation:

\[ D \lesssim \frac{6}{(2\pi T)} \]
Theoretical Motivation:

The quark doesn’t move. Where is the noise in “standard” AdS/CFT?
Moving Quarks in AdS/CFT: (HKKKY; S. Gubser)

1. No transverse acceleration!

2. No photon emission for example

Need to find the noise in AdS/CFT
Langevin description of heavy quark thermalization:

- Write down an equation of motion for the heavy quarks.

\[
\frac{dx}{dt} = \frac{p}{M} \\
\frac{dp}{dt} = -\eta_D p + \xi(t)
\]

Drag \hspace{1cm} Random Force

- The drag and the random force are related

\[
\langle \xi_i(t)\xi_j(t') \rangle = \frac{\kappa}{3} \delta_{ij} \delta(t - t') \\
\eta_D = \frac{\kappa}{2MT}
\]

\( \kappa = \text{Mean Squared Momentum Transfer per Time} \)

- People computed the coefficients \( \kappa \) and \( \eta \)

Want to see the whole brownian process!
Langevin in Quantum Mechanics

\[ M \frac{d^2 x}{dt^2} + \frac{\kappa}{2T} \dot{x} = \xi \]

- Consider a heavy particle coupled to bath a force on the contour

\[ Z_Q = \left\langle \int Dx_1 Dx_2 e^{i \int \frac{1}{2} M v_1^2 - i \int \frac{1}{2} M v_2^2} e^{i \int dt_1 F_1 x_1} e^{-i \int dt_2 F_2 x_2} \right\rangle \text{Bath} \]

- The force term is small compared to the inertia

\[ \left\langle e^{i \int dt_1 F_1 x_1} e^{-i \int dt_2 F_2 x_2} \right\rangle_{\text{bath}} \sim e^{-\frac{1}{2} \int dt \int dt' x_a(t) \langle F_a(t) F_b(t') \rangle x_b(t')} \]

- Stir the soup:
  - Now switch vars to ave and diff: \( \bar{X} = (x_1 + x_2)/2 \) \( \Delta X = x_1 - x_2 \)
  - Do the path integral over difference
Result Generalized Langevin

\[ M_Q \frac{d^2 \bar{X}}{dt^2} + \int_{t'}^t G_R(t - t') \bar{X}(t') = \xi \]

1. Drag = retarded force force correlator

\[ G_R(t) = \theta(t) \langle [F(t), F(0)] \rangle \]

2. Noise = symmetrized force-force correlator

\[ \langle \xi(t) \xi(0) \rangle = \langle \{F(t), F(0)\} \rangle \]
AdS/CFT in the Kruskal Plane

- Source fields for "1" and "2" operators live on the right and left quadrants

\[
\mathcal{O}_1, \mathcal{O}_2 \leftrightarrow \phi_1, \phi_2
\]

\[
\left\langle e^{i \int dt_1 \phi_1 \mathcal{O}_1} e^{-i \int dt_2 \phi_2 \mathcal{O}_2} \right\rangle_{SYM} = e^{iS[\phi_1, \phi_2]}
\]
\[(t, r)\] Observer vs. Kruskall Observer

**Behind Event Horizon**
Integrating out the Bulk

- The real time partition function of string for small fluctuations

\[ Z = \int \prod_{t_1} d\mathbf{X}_1(t_1) \prod_{t_2} d\mathbf{X}_2(t_2) \prod_{t,z} d\mathbf{x}_1(t,z) d\mathbf{x}_2(t,z) e^{iS_{NG}} \]

- The integrals over the internal coordinates can be done and yield

\[ Z = \int D\mathbf{X}_1 D\mathbf{X}_2 e^{iS_{eff}[X_{cl}(\mathbf{X}_1(t_1), \mathbf{X}_2(t_2))]} \]
Strings and Fluctuations

\[ S_{NG, \text{Left}} \sim \frac{R^2}{2\pi \ell_s^2} \int dt \, dr \left[ 1 - \frac{1}{2} \left( \frac{\dot{x}_\parallel^2}{f} - 4fr^2 \left( x'_\parallel \right)^2 \right) \right] \]

**Quadratic Fluctuations**
Strings and Path Integrals

\[ Z_{str} = \int \prod_{t_1} dX_1(t_1) \prod_{t_2} dX_2(t_2) \prod_{t,z} dx_1(t,z) dx_2(t,z) e^{iS_{NG}} \]

\[ = \int D X_1 D X_2 e^{iS_{eff}[X_{cl}(X_1(t_1),X_2(t_2))]} \]
The effective action

\[ iS_{\text{eff}} = \left( -\frac{1}{2} \int \frac{d\omega}{2\pi} \right) \]
\[ + X_1(-\omega) \left[ -iM_Q^0\omega^2 + \langle F_1F_1 \rangle \right] X_1(\omega) \]
\[ + X_2(-\omega) \left[ +iM_Q^0\omega^2 + \langle F_2F_2 \rangle \right] X_2(\omega) \]
\[ - X_1(-\omega) \left[ \langle F_1F_2 \rangle \right] X_2(\omega) \]
\[ - X_2(-\omega) \left[ \langle F_2F_1 \rangle \right] X_1(\omega) \]

where for example

\[ \langle F_1F_1 \rangle (\omega) = \text{Force-Force Correlator in 1} \]
\[ \langle F_2F_2 \rangle (\omega) = \text{Force-Force Correlator in 2} \ldots \]
\[ \langle F_1F_2 \rangle (\omega) = \text{Force-Force cross correlator} \ldots \]

Same as in Quantum Mechanics...
Result: Langevin with Memory

- Find the endpoint of the string obeys the expected Langevin equation

\[ M_Q^0 \frac{d^2X}{dt^2} + \int_{t'}^t G_R(t - t')X(t') = \xi \]

- To quadratic order the retarded green function is

\[ G_R(\omega) = (\Delta M)\omega^2 - i\omega \left( \frac{\kappa}{2T} \right) \]

\[ \sqrt{\lambda T}/2 \quad \kappa=\sqrt{\lambda\pi T^3} \]

- Then find the following effective equation of motion

\[ M_{\text{kin}}(T) \frac{d^2X}{dt^2} + \kappa \frac{dX}{2Td} = \xi \]

with the kinetic mass (Herzog et al ’06)

\[ M_{\text{kin}}(T) = M_Q^0 - \frac{\sqrt{\lambda T}}{2} \]
Phenomenological Summary

\[ D = \frac{2T^2}{\kappa} = \frac{2}{\sqrt{\lambda \pi T}} \]

- Best QCD estimate from \( \mathcal{N} = 4 \)
  \[ D_{QCD} \sim \frac{4 \div 8}{2\pi T} \]

- Weak Coupling best estimate
  \[ D_{QCD} \sim \frac{3 \div 6}{2\pi T} \]
Generalize to Relativistic Heavy Quarks

- Transverse Momentum Broadening of a heavy quark (analogous to $\hat{q}$)

\[
\kappa_T(v) = \text{Mean squared transverse momentum transfer per unit time}
\]

\[
\kappa_L(v) = \text{Mean squared longitudinal momentum transfer per unit time}
\]

- Drag

\[
\frac{dP}{dt} = -\eta(v)P + \xi_L(t) + \xi_T(t)
\]
Finding the semi-classical string (Herzog et al and S. Gubser)

- Turn on an electric field to accelerate the quark

- A semiclassical string trails behind the quark

\[
x_3 = vt + \frac{v}{2} \left[ \arctan(z) - \text{arctanh}(z) \right]
\]
Quantum Mechanics of the Endpoint

- There is a radius where the string exceeds the local speed of light

\[ r_{\text{critical}} = \sqrt{\gamma r_o} \]

- Analogy with black holes can be made precise
Strings and Fluctuations

Left Universe

Force $F_2(t)$

$X_2(t)$
Left Boundary

$\sqrt{\gamma r_H}$

Right Universe

Force $F_1(t)$

$X_1(t)$
Right Boundary
Effective Equation Motion

\[ M_{\text{kin}}(T) \frac{d(\gamma \mathbf{v})}{dt} = -\frac{\sqrt{\lambda \pi T^2}}{2} \gamma \mathbf{v} + \xi^i(v) \]

\begin{align*}
\text{Drag} & \quad \text{Noise} \\
\end{align*}

- Drag grows \( \gamma \) – Relaxation time independent of momentum \( p = p_0 e^{-\eta t} \)

- Fluctuations also grow with momentum

\[ \kappa_T(v) = \sqrt{\lambda \pi} T^3 \times \sqrt{\gamma} \]
\[ \kappa_L(v) = \sqrt{\lambda \pi} T^3 \times \gamma^{5/2} \leftarrow \text{(Gubser)} \]

- Effective Mass decreases with gamma

\[ M_{\text{kin}}(T) = M_0^0 - \frac{\sqrt{\lambda T}}{2} \times \sqrt{\gamma} \]

Constraint on velocity: \( \gamma \ll M_Q / \sqrt{\lambda T} \)
Consequence of velocity constraint: No LPM

- Case 1: (Dead Cone) Photon decoheres because it moves faster than quark

\[ t_{\text{decoh-v}} \sim \frac{1}{\omega(1 - v \cos(\theta))} \sim \frac{\gamma^2}{\omega} \]

- Case 2: (LPM) Photon decoheres due to transverse momentum broadening

\[ t_{\text{decoh-LPM}} \sim \frac{E}{\sqrt{\hat{q}} \omega} \]

- But the quark stops in a finite time independent of momentum.

\[ t_{\text{stop}} \sim \frac{M}{\sqrt{\lambda T^2}} \]

- To see the LPM need

\[ t_{\text{decoh-LPM}} \ll t_{\text{decoh-v}} \ll t_{\text{stop}} \]

These conditions and the velocity constraint can’t be simultaneously satisfied
Conclusions

- Quantum Mechanics of $AdS_5$ leads to thermal noise
  - Prototypical Example – Brownian Motion
  - Other examples – “Long Time” hydrodynamic tails
  - Necessary for thermalization?

Order of limits Matters!

\[
\begin{cases}
\text{Time, } N_c \rightarrow \infty \\
\text{Time, } \sqrt{\lambda} \rightarrow \infty
\end{cases}
\]

- Saw some applications of $AdS$ to heavy quark data
  - Thermal perturbation theory is poor. Quark and Gluons as Quasi-Particles?
  - $AdS$ predictions are markedly different from perturbation theory.

  More likely wrong than right! But maybe we should find out for sure