Baryonic Spectral Functions at Finite Temperature

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QCD Phase Diagram

Hadron Phase
- chiral symmetry breaking
- confinement

1st order crossover

CEP (critical end point)

QGP (quark-gluon plasma)

CSC (color superconductivity)

LHC

RHIC

160-190 MeV
100 MeV ~ $10^{12}$ K

$\mu_B$ 5-10$\rho_0$

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Hadrons above $T_c$?

- No *a priori* reason that no hadrons exist above $T_c$
- QGP looks like strongly interacting system (low viscosity...etc.)

**Definition of Spectral Function (SPF)**

$$
\rho_{\mu\nu}(k_0, \vec{k}) = \frac{1}{(2\pi)^3} \sum_{n,m} \frac{e^{-(E_n - mN_\mu)/T}}{Z} \langle n | J_\mu(0) | m \rangle \langle m | J_\nu(0) | n \rangle (1 \pm e^{-P^\mu_n/T}) S^4(k - P_{mn})
$$

$J_\mu(0)$: A Heisenberg Operator with some quantum #

$|n\rangle$: Eigenstate with 4-momentum $P^\mu_n$

$P_{mn} = P_m - P_n$

**Information on**
- Dilepton production
- Photon Production
- $J/\psi$ suppression...etc.: encoded in SPF

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Microscopic Understanding of QGP

Importance of Microscopic Properties of matter, in addition to Bulk Properties

- In condensed matter physics, common to start from one particle states, then proceed to two, three, ... particle states (correlations)

Spectral Functions:

- One Quark — need to fix gauge
- Two Quarks
  - mesons
    - color singlet
    - octet — need to fix gauge
  - diquarks — need to fix gauge
- Three Quarks
  - baryons
- ......
Photon and Dilepton production rates

- Photon production rate

\[
\frac{d^3 R_\gamma}{d^3 p} = \frac{\alpha}{\pi} \rho_T(\omega = |\vec{p}|, |\vec{p}|) \exp(\omega/T) - 1
\]

- Dilepton production rate

\[
\frac{d^4 R_{ll}}{d^4 p} = \frac{\alpha^2}{3\pi^2 p^2} (2\rho_T + \rho_L)(\omega, \vec{p}) \exp(\omega/T) - 1
\] (for massless leptons)

where \(\rho_T\) and \(\rho_L\) are given by

\[
\rho_{\mu\nu}(\omega, \vec{p}) = \rho_T(\omega, \vec{p})(P_T)_{\mu\nu} + \rho_L(\omega, \vec{p})(P_L)_{\mu\nu}
\]

\(\rho_{\mu\nu}\): QCD EM current spectral function

\[
\rho_{\mu\nu}(k_0, \vec{k}) = \rho_T(k_0, \vec{k})(P_T)_{\mu\nu} + \rho_L(k_0, \vec{k})(P_L)_{\mu\nu}
\]

\[
J_\mu = J^\text{em}_\mu = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s + \ldots
\]
Lattice calculation of spectral functions

- No calculation yet for finite momentum light quark spectral functions
  \[
  \frac{d^3 R_\gamma}{d^3 p} = \frac{\alpha \rho_T(\omega = |\vec{p}|, \vec{p})}{\pi \exp(\omega/T) - 1}
  \]

  LQGP collaboration, in progress

  So far, only pQCD spectral function has been used

- Calculation for zero momentum light quark spectral functions by two groups
  \[
  \frac{d^4 R_{\pi\pi}}{d^4 p} = \frac{\alpha^2}{3 \pi^2 p^2} \frac{(2 \rho_T + \rho_L)(\omega, \vec{p})}{\exp(\omega/T) - 1}
  \]

  \[\rho_T = \rho_L \text{ @ zero momentum}\]

  Since the spectral function is a real time quantity, necessary to use MEM (Maximum Entropy Method)
QGP is strongly coupled, but...

Direct $\gamma$ spectra: hydro+pQCD prediction (LHC)

Photon spectra: hydro + (quenched) NLO pQCD:

Pb-Pb 0-10% central ($<b> \sim 3$ fm)

Pb-Pb 70-90% periph. ($<b> \sim 14$ fm)

Last Call Hi-LHC, May 29th, 2007
Lattice calculation of spectral functions

- No calculation yet for finite momentum light quark spectral functions

\[ \omega \frac{d^3 R_{\gamma}}{d^3 p} = \frac{\alpha}{\pi} \rho_\gamma(\omega = |\vec{p}|, \vec{p}) \exp(\omega/T - 1) \]

LQGP collaboration, in progress

So far, only pQCD spectral function has been used

- Calculation for zero momentum light quark spectral functions by two groups

\[ \frac{d^4 R_{\gamma\gamma}}{d^4 p} = \frac{\alpha^2}{3\pi^2 p^2} \frac{(2\rho_\gamma + \rho_L)(\omega, \vec{p})}{\exp(\omega/T - 1)} \]

\[ \rho_\gamma = \rho_L @ zero momentum \]

Since the spectral function is a real time quantity, necessary to use MEM (Maximum Entropy Method)
Spectral Functions above $T_c$

$m \sim m_s$

Lattice Artifact

$A(\omega)/\omega^2$

$\omega$ [GeV]

peaks structure
Another calculation

In almost all dilepton calculations from QGP, pQCD expression has been used.

How about for heavy quarks?

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$J/\psi$ non-dissociation above $T_c$

$Lattice Artifact$

$J/\psi (p = 0)$ disappears between $1.62 T_c$ and $1.70 T_c$

Asakawa and Hatsuda, PRL 2004

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Result for PS channel ($\eta_c$) at Finite $T$

$\eta_c$ ($p = 0$) also disappears between $1.62T_c$ and $1.70T_c$
Importance of Understanding Baryons @ Finite T

Success of Recombination @ RHIC

\[ v_2^M(p_t) \sim 2v_2^p\left(\frac{p_t}{2}\right) \quad \text{and} \quad v_2^B(p_t) \sim 3v_2^p\left(\frac{p_t}{3}\right) \]

\[(1 + x)^n \sim 1 + nx\]

\[ v_2^M(p_T^M) : v_2^B(p_T^B) \sim 2 : 3 \quad \text{for} \quad p_T^M : p_T^B = 2 : 3 \]

\[ \frac{dN_s}{dyd\varphi} \left(\frac{dN_s}{dyd\varphi d^2p_T}\right) = N_{s0}\left(1 + 2v_1 \cos(\varphi - \varphi_0) + 2v_2 \cos2(\varphi - \varphi_0) + \cdots\right) \]
**Constituent Quark Number Scaling**

- \( v_2^M(p_T^M) : v_2^B(p_T^B) \sim 2 : 3 \) for \( p_T^M : p_T^B = 2 : 3 \)

- Partons are flowing and Partons recombine to make mesons and baryons

**Assumption**

*All hadrons are created at hadronization simultaneously*

\[ n=2 \quad \pi^+ + \pi^- \quad PHENIX \]

\[ n=3 \quad p + \bar{p} \quad PHENIX \]

\[ K^+ + K^- \quad PHENIX \]

\[ K^0 \quad STAR \]

\[ \Lambda + \bar{\Lambda} \quad STAR \]

\[ \Xi + \bar{\Xi} \quad STAR \]
Baryon Operators

- Nucleon current

\[ J_N(x) = \epsilon_{abc} \left[ s \left( u_a(x)Cd_b(x) \right) \gamma_5 u_c(x) + t \left( u_a(x)C\gamma_5d_b(x) \right) u_c(x) \right] \]

\[ s = -t = 1 \text{ Ioffe current} \]

- On the lattice, used \( s = 0, t = 1, u(x) = d(x) = q(x), J_N(x) \rightarrow J(x) \)

- Euclidean correlation function at zero momentum

\[ D(\tau,0) = \int d^3x \left\langle J(\tau,\vec{x})J(0,\vec{0}) \right\rangle \]

\[ D(\tau,0) = \int_{-\infty}^{\infty} K(\tau,\omega)\rho(\omega)d\omega \]

\[ K(\tau,\omega) = \frac{\exp\left(\frac{1}{2}-\tau T\right)\frac{\omega}{T}}{\exp\left(\frac{\omega}{2T}\right) + \exp\left(-\frac{\omega}{2T}\right)} \]
\[ D(\tau, 0) = \int d^3 x \left\langle J(\tau, \vec{x}) J(0, \vec{0}) \right\rangle \]

\[ D(\tau, 0) = \int_{-\infty}^{\infty} K(\tau, \omega) \rho(\omega) d\omega \]

\[ \rho(\omega) = \rho_0(\omega) \gamma^0 + \rho_s(\omega): \quad \rho_0(\omega), \rho_s(\omega) \text{ independent} \]

\[ = \rho_+ (\omega) \Lambda_+ \gamma^0 + \rho_- (\omega) \Lambda_- \gamma^0 \]

\[ \rho_0(\omega) = \rho_0(-\omega), \quad \rho_s(\omega) = -\rho_s(-\omega) \]

\[ \rho_+ (\omega) = \rho_- (-\omega) = \rho_0(\omega) + \rho_+ (\omega) \geq 0 \quad \text{semi-positivity} \]

\[ \rho_+ (\omega) (\rho_- (\omega)): \text{ neither even nor odd} \]

Thus, need to and can carry out MEM analysis in \([-\omega_{\text{max}}, \omega_{\text{max}}]\)

In the following, we analyze \( \rho(\omega) \equiv \frac{\rho_+ (\omega)}{|\omega^5|} \)
1. Lattice Sizes
   \[ 32^3 \times 46 \ (T = 1.62 T_c) \]
   \[ 54 \ (T = 1.38 T_c) \]
   \[ 72 \ (T = 1.04 T_c) \]
   \[ 80 \ (T = 0.93 T_c) \]
   \[ 96 \ (T = 0.78 T_c) \]

2. \( \beta = 7.0, \ \xi_0 = 3.5 \)
   \[ \xi = a_\sigma / a_\tau = 4.0 \text{ (anisotropic)} \]

3. \( a_\tau = 9.75 \times 10^{-3} \text{ fm} \)
   \[ L_\sigma = 1.25 \text{ fm} \]

4. Standard Plaquette Action

5. Wilson Fermion

6. Heatbath : Overrelaxation
   \[ = 1 : 4 \]
   1000 sweeps between measurements

7. Quenched Approximation

8. Gauge Unfixed

9. \( \mathbf{p} = \mathbf{0} \) Projection

10. Machine: CP-PACS
Analysis Details

- Default Model

At zero momentum,

\[
\rho_+ (\omega) = \rho_- (\omega) = \frac{1}{(2\pi)^4} \frac{5}{128} \text{sgn}(\omega) \omega^5
\]

Espriu, Pascual, Tarrach, 1983

- Relation between lattice and continuum currents

\[
J^\text{LAT} (\tau, \vec{x}) = a_t^{3/4} a_\sigma^{15/4} \left( \frac{1}{2\sqrt{K_t K_\sigma}} \right)^{3/2} \frac{1}{Z_O} J^\text{CON} (\tau, \vec{x})
\]

- In the following, lattice spectral functions are presented
- \( Z_O = 1 \) is assumed

- \( \omega_{\text{max}} = 45 \text{ GeV} \sim \frac{3\pi}{a_\sigma} \) (3 quarks)
Stat. and Syst. Error Analyses in MEM

Generally,

The Larger the Number of Data Points and the Lower the Noise Level

The closer the result is to the original image

Need to do the following:

- Put Error Bars and Make Sure Observed Structures are Statistically Significant
- Change the Number of Data Points and Make Sure the Result does not Change

in any MEM analysis

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Below $T_c$: Light Baryon

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Below $T_c$: Charm Baryon

$\rho(\omega)$

$T=0.78T_c$
ccc channel

ccc baryon

parity -
parity +

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Above $T_c$: Light Baryon

peak near zero + symmetric and equally separated peaks

$T = 1.38 T_c$

sss channel

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only symmetric and equally separated peaks

$T = 1.62T_c$

sss channel

parity -

parity +

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Above $T_c$: Charm Baryon

peak near zero + symmetric and equally separated peaks

$\rho(\omega)$

$T=1.38T_c$
ccc channel

parity -
parity +

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@Higher $T$

only symmetric and equally separated peaks

$\rho(\omega)$

$T=1.62T_c$

ccc channel

$\omega$ [GeV]

parity -

parity +

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Statistical Analysis: Light Baryon @ T=0

Peaks are statistically significant

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Statistical Analysis: Charm Baryon @Finite T

Peak near zero is statistically significant

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Origin of Near Zero Structure

Scattering Term

- Scattering term at $\vec{p} = \vec{0}$ a.k.a. Landau damping

\[ J_N \]

\[ \vec{p} = \vec{0} \]

- This term is non-vanishing only for

\[ 0 < |\omega| = |\omega_2 - \omega_1| \leq |m_2 - m_1| \]

- For $J/\psi$ ($m_1 = m_2$), this condition becomes

\[ 0 < |\omega| = |\omega_2 - \omega_1| \leq \epsilon \quad \text{zero mode} \]

cf. QCD SR (Hatsuda and Lee, 1992)
Scattering Term (two body case)

\[ m_1 (\omega_1, \vec{p}_1) \times \]
\[ J_N \]
\[ \vec{p} = 0 \]
\[ m_2 (\omega_2, \vec{p}_1) \]

- This term is non-vanishing only for 0 < |\omega| = |\omega_2 - \omega_1| \leq |m_2 - m_1|

(Boson-Fermion case, e.g. Kitazawa et al., 2008)

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Scattering Term (three body case)

$m_1(\omega_1, \vec{p}_1)$

$m_2(\omega_2, \vec{p}_2)$

$m_3(\omega_3, \vec{p}_3)$

$J_N$

$\vec{p} = 0$

$T \ll m_1, m_2, m_3$

$|\vec{p}_1| \sim 0$

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Negative parity: a possible interpretation

- Anti-quark: parity -

\[ m_1 \otimes m_2 \]

\[ (\omega_1, \vec{p}_1) \quad \text{if } 0^+ \text{ or } 1^+ \]

\[ J_N \quad \vec{p} = 0 \]

\[ L \sim 0 \]

\[ m_2 \]

\[ (\omega_2, \vec{p}_1) \]

Then: parity –

\[ T \ll m_1, m_2 \]

\[ |\vec{p}_1| \sim 0 \]

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Origin of Symmetric Structure

- Wilson Doublers

Mass of Wilson Doublers with \( r = 1 \) in the continuum limit

\[ m + \frac{2n_\pi}{a} \]

\( n_\pi \): number of momentum components equal to \( \pi \) (1,2,3)

- If quark mass can be neglected:
  - Masses of baryons with doublers
  - Scattering term peaks with quark-doubler, doubler-doubler pairs

\[ \text{Approximately equally separated and symmetric in } \omega \]
QCD Phase Diagram

- Quark-antiquark correlations
- Diquark correlations
- Baryon correlations
- QGP (quark-gluon plasma)

Hadron Phase
- chiral symmetry breaking
- confinement

CSC (color superconductivity)

160-190 MeV
100 MeV ~ $10^{12}$ K

LHC
RHIC
CEP (critical end point)

order?
Summary

- Baryons disappear just above $T_c$

- A sharp peak with negative parity near $\omega=0$ is observed in baryonic SPF above $T_c$
  This can be due to diquark-quark scattering term and imply the existence of diquark correlation above $T_c$

- Diquarks disappear below meson disappearance temperature

- Direct measurement of SPF of one and two quark operators with MEM is desired

- To understand doubler contribution, calculation with finer lattice is desired