Determining Your Position

The determination of your position on the Earth by measuring the position of celestial objects is known as celestial navigation. The topic of celestial navigation is very large, and we will only scratch the surface here.

Special note: Actual, accurate, celestial navigation requires attention to many more details than presented here.

In real life, mistakes in navigation may result in your death or the destruction of your vessel. This page discusses the very fundamentals from an astronomy viewpoint, but if you are actually going to rely on navigation in real life, sailing perhaps, you should take navigation courses, to include piloting, GPS and radio navigation, and celestial navigation. Some excellent navigation courses (but not celestial) are available at modest charge taught by the US Coast Guard Auxiliary (go to www.cgaux.org for more information.)

Longitude

At any moment there is one spot on the Earth where the sun is directly overhead. This spot, called the subsolar point, moves around the Earth in one solar day, 24 hours, thus the subsolar point moves 15° of longitude for each hour of the day (360° / 24h).
Since the subsolar point moves 15° per hour, if we knew the difference in time between the transit of the sun (and hence the subsolar point) on any specific meridian and the transit of the sun for our local meridian, we could use this to determine our longitude. Here’s how it works:

On a specific day we determine that the sun crossed our meridian at 15h 30m Universal Time (UT). We can find in an ephemeris (a table of astronomical positions and times) that on this day the sun transited the Greenwich meridian (0° longitude) at 12:00 UT. The difference in the times of transit is

$$15.5^h - 12.0^h = 3.5^h$$

We know the sun moves 15°/h, so in 3.5 hours the sun will have moved

$$15^\circ/h \times 3.5h = 52.5^\circ$$

So our longitude is 52.5° from the longitude of Greenwich (which is 0°, of course.) But are we 52.5° to the east of Greenwich, or are we 52.5° to the west of Greenwich? Since the sun moves from east to west, the sun will transit points to the east of us at an earlier time than it transits our meridian. The sun transited Greenwich at 12:00 UT, and our meridian at 15:30 UT, so we must be west of Greenwich. Our longitude then is 52.5° west.

Mathematically, we can express the method for determining our longitude as:

$$\lambda = 15^\circ (L_T - G_T)$$

Where \(L_T\) is the UT of solar transit of our local meridian, \(G_T\) is the UT of solar transit at Greenwich, and \(\lambda\) is our longitude. A negative value for \(\lambda\) indicates East longitude, a positive value is West longitude.

Latitude
There are several ways to determine your latitude from celestial observations. Make sure you don’t confuse these similar methods. Be especially careful that you completely understand the terms altitude, declination, and latitude.

Method 1) The easiest way is to recall that the declination of a star at your zenith is equal to your latitude. Using a star chart you determine which star is directly above you. Find the declination of that star in an ephemeris or star catalogue. The declination is your latitude.

Methods 2 and 3) The altitude of the celestial pole is equal to your latitude.

There are two ways to find the altitude of the celestial pole.

a) Measure the altitude of the north star, Polaris. (For practical purposes in this class we will assume that Polaris is exactly at the north celestial pole, but in reality it is very slightly off.) Having made your measurement of the pole’s altitude, you know your latitude, because the altitude of the celestial pole is equal to your latitude.
Mathematically, this is blindingly straightforward:

\[ L = A_p \]

Where \( L \) is our latitude and \( A_p \) is the altitude of the pole. Unfortunately, there is no star close enough to the south celestial pole to be used like Polaris, so in the southern hemisphere we have to use the next way to find the altitude of the celestial pole.

**b)** Find the declination of a star just barely touching the horizon, either dead north of you (in the northern hemisphere) or dead south of you (in the southern hemisphere) that is the minimum declination that a star and have and still be in the circumpolar zone as seen from your location.

What we need to know is the altitude of the celestial pole. The altitude is the *angle* between the horizon and the pole. If we know the minimum declination that is circumpolar, we can subtract that from the declination of the celestial pole,
which is 90° N or S, which gives us the angle between the horizon and the pole. That angle is our latitude.

Mathematically, we can write:

\[ L = 90° - \delta_m \]

Where \( L \) is our latitude and \( \delta_m \) is the minimum circumpolar declination.

**Method 4)** Determine the difference in latitude between you and the subsolar point. If we know the subsolar point is at latitude 22° north, and we know our latitude is 30° north of the subsolar point, we simply add them and find our latitude is 52° north.

To find the difference in latitude between us and the subsolar point, we measure the altitude of the sun exactly as it transits. If the sun were directly overhead at the zenith, it would have an altitude of 90°. If we were 90° away from the subsolar point, we’d measure the sun’s altitude as 0°, it would just be on our horizon. **At solar transit, for every degree that the sun’s altitude is less than 90°, we are 1° of latitude away from the subsolar point.** Mathematically we can express this as

\[ \Delta L = 90° - A_s \]

Where \( \Delta L \) is the difference in latitude between our latitude and the subsolar point, and \( A_s \) is the altitude of the sun at transit.

So now we look up the sun’s declination for this day. Remembering that the latitude of a spot on the Earth is equal to the declination of a celestial object at the zenith, we know that the sun’s declination is the latitude of the subsolar point.

Now we have almost all we need. We also need to note if the sun at transit was north or south of us. Our latitude is the declination of the sun plus the difference
in the latitudes of the subsolar point and our position. We use negative declination values for southern declination and positive for northern declination. We also use negative values for $\Delta L$ if we are south of the sun (the sun is north of us at transit) and positive $\Delta L$ values if we are north of the sun (the sun was south of us at transit). Mathematically,

$$L = \delta_s + \Delta L$$

Where $L$ is our latitude, $\delta_s$ is the declination of the sun, and $\Delta L$ is the difference in latitude between our position and the position of the subsolar point.

**Putting it together – The noon sight**

Combining this method 4 for finding latitude with the method of finding the longitude described earlier is a classic technique in celestial navigation known as a “noon sight”. A noon sight allows us to determine our latitude and longitude from a single measurement of the sun’s altitude and the time of transit. In practice it is not quite as simple as this. For example, there are several corrections to our measured value of the sun’s altitude that must be applied (refraction, dip of the horizon, and index error) to give us the true altitude of the sun, and adjustments to the measured time of transit, and to the listed declination values. Nonetheless, the astronomy part of a real noon sight is exactly what we have discussed.

**Position other than using a noon sight**

(Supplemental information, not required for PY124 students)

You may ask: “What if it is cloudy at noon? You can’t measure the sun’s altitude if you can’t see the sun. What other methods are there for determining latitude and longitude from astronomical measurements?”
The fact is that although the noon sight is by far the simplest method of celestial navigation, its unfortunate property of only being useful for one moment each day (exactly local noon) means that in celestial navigation other methods are usually employed to determine position.

It is possible to determine our position with two measurements of the altitude of the sun, with no restrictions on the position of the sun at those times, except that the time of the two measurements must be separated by at least 30 minutes. We can also do the same measurement with certain stars, or the planets. The mathematics to do these methods are quite involved, and require a knowledge of spherical geometry and spherical trigonometry. (Spherical geometry applies to the geometry of the surface of a sphere, rather than of a flat surface as in Euclidean (“normal”) geometry. The rules are all different, for example in spherical geometry parallel line can meet, and you can have a triangle with three 90° angles. Almost no one except astronomers uses it.)

Luckily for the world’s navigators, the practicing navigator uses a set of books with tables of altitudes, positions, and corrections, along with a standard form that allows the navigator to do a small number of addition and subtraction operations with various data from the tables to get the vessel’s position.

Of course, today celestial navigation serves only as an emergency back up to electronic and satellite navigation, which allow the navigator to push a button and have his latitude and longitude displayed instantly. Some navigation apparatus is available that will actually show a nautical chart of the area near the vessel on a computer screen, with a mark showing exactly where the vessel is located, along with the vessel’s speed and direction of motion.

But that’s just not as much fun as celestial navigation. (At least to astronomer!)